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An Investigation of the Effects of Crack on the Zone of Pull-in Suppression in Micro-Electromechanical Systems Using High-Frequency Excitation

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ABSTRACT: In this paper, the pull-in phenomenon is suppressed using a range of values of amplitude and frequency of high-frequency voltage excitations in the post pull-in condition of the cracked micro-electromechanical systems. These specified ranges are named as stable zones. It is investigated the effects of the crack parameters (depth and location) on changes of these zones, in the post pull-in condition. It is shown that these zones have different areas for different crack parameters. The cracked micro-beam is modeled as a single-degree-of-freedom systems consist of mass-spring-damper and the motion equation of the cracked micro-beam is extracted. The method of direct partition of motion is used to split the fast and slow dynamics. By means of slow dynamic part, the effects of the crack on the averaged position of vibration of cracked micro-beam are investigated versus voltage amplitude and frequency of the high-frequency AC. By approaching the crack to the fixed end or increasing the depth of crack, the stability zone reduced. Therefore, the pull-in instability can be suppressed in the lower range of amplitude and frequency. This method can be used in sensors' health-monitoring and one can predict the parameters of the crack using this method.

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1- Introduction

By developing technology, micro-electromechanical systems (MEMS) and structures are increasingly used in various techniques such as astronavigation, radars, telecommunication, personal mobile phones, and biotechnology. Capacitive micro-switches [1, 2] and resonant micro-sensors [3, 4] and pressure sensor are some of these systems.

There are a number of actuation methods such as thermal, pneumatically and piezoelectric excitation for MEMS devices. Electrostatic actuation is the most well-established of this actuation method because of its simplicity and high efficiency [5]. Most of the micro-electromechanical structures contain movable micro-beam suspended above a stationary electrode and connected to a voltage source. The electric loads are composed of a DC polarization voltage and an AC voltage. In the resonators, the DC component applies an electrostatic force on the micro-beam and thereby deflecting it to a new equilibrium position, while the AC component vibrates the micro-beam around this equilibrium position [6]. The studies show that the nonlinear behavior of micro-beams is the result of the reaction of several forces such as electrostatic force and mechanical restoring force, which are considered as nonlinear terms in the governing equation. The mechanical restoring force is arising from residual, axial and fringing stress [7]. The elastic forces grow about linearly with displacement whereas the electrostatic forces grow inversely proportional to the square of the distance between movable micro-beam and the stationary electrode. When the excitation voltage increases, the balance of the forces is upset and the electric force becomes more than mechanical restoring force and it makes the micro-beam collapse and hence the failure of the device. This structural instability is known as pull-in phenomenon and the corresponding voltage is considered as pull-in voltage. This phenomenon is one of the most significant challenges in designing MEMS. The importance of knowing the exact amount of pull-in voltage, especially in capacitive structures, is the determination of material characteristics such as Young modulus and residual stress and the stiffness of structure [8]. Many studies have been conducted on the field of pull-in phenomenon and the effect of various factors on its value. Also, the special tools are provided to predict the pull-in occurrence in order to design the MEMS device in the stable zone and far from pull-in phenomenon [9, 10]. Zhang and Zhao [11] studied the effect of nonlinear factors on the pull-in instability of microstructure under electrostatic loading, using the combination of Galerkin method and Cardan solution of the cubic equation. They used Taylor series to expand the electrostatic loading term in the one-mode analysis method. Rezazadeh et al. [7] extracted the static pull-in voltage of two elastic parallel fixed-fixed and cantilever micro-beams in MEMS under nonlinear effects such as residual stresses, fringing field, and axial stresses. In this research, instead of a beam parallel with fixed electrode, two parallel microbeams are embedded in the micro-structures. The Step-by-Step Linearization Method (SSLM) was used. They showed that the percentage of pull-in voltage reduction, when compared these micro-beams to simple fixed-fixed and cantilever models, is of the order of 27-30%. Mojahedi et al. [12] investigated static pull-in instability of electrostatically-actuated micro-bridges and micro- cantilevers and the effects of different nonlinear factors on static pull-in behavior. They utilized Galerkin's decomposition method to convert the nonlinear differential equations of motion to nonlinear

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integro-algebraic equations and used homotopy perturbation method.

In many types of research, the stability of static deflection of micro-beams was investigated and there are only a few studies about dynamic stability around equilibrium under AC voltage. But the MEMS devices are often dynamically excited and the research shows that there is a possibility for a dynamic instability to trigger pull-in below the statically predicted instability limit. Therefore, the analysis of the behavior of micro-beams under the AC loading is important and necessary because of the nonlinear behavior of MEMS systems. In this regard, Nayfeh et al. [13] investigated the dynamic pull-in instability and pull-in phenomenon characteristics in the micro-electromechanical resonators in the presence of combination of AC and DC. Also, they formulated safety criteria for the design of MEMS resonant sensors and filters excited near one of their natural frequency. According to their study, there is a possibility of dynamic pull-in phenomenon at voltages less than the static voltage and three distinct mechanisms leading to the dynamic pull-in instability. Sadeghi et al. [14] studied the effects of the size dependence of the static and dynamic behavior of the electrostatically actuated micro-beams under nonlinear electrostatic load. In their research, it was shown size-dependent behavior of materials appears for a structure when the characteristic size such as thickness or diameter is close to its internal length-scale parameter. They used the step-by-step linearization method and reduced order model based on Galerkin, respectively, to solve static and dynamic equations. According to their studies, there is a considerable difference between the static and dynamic pull-in voltages gained using the classic beam theory and the modified couple stress theory and also using the modified couple stress theory decreases the difference with the experimental results.

The pull-in instability is important and a sensitive phenomenon in micro-electromechanical devices. Therefore, one of the interesting topics of researchers is to provide the method and devices for the suppression of pull-in phenomenon and stabilization of micro-electromechanical systems in the post pull-in state. In this regard, Lakrad and Belhaq [15] tried to suppress the electrostatically induced pull-in instability in the electro-mechanical systems using a high-frequency voltage. They modelled the microstructure as a single-degree-of-freedom system and investigated the effect of the high-frequency voltage on the pull-in instability in the intact micro-structures. They also provided a method to suppress the electrostatically induced pull-in instability for a range of values of the amplitude and the frequency of the high-frequency AC. Lakrad and Belhaq [16] in another research showed that it is possible to suppress the pull-in phenomenon, using high-frequency AC voltage, in a microstructure actuated by mechanical shocks and electrostatic forces in the post pull-in state.

There is the possibility of failures such as crack, during manufacturing and performance of MEMS. These failures may lead to change the stability threshold and early pull-in phenomenon occurrence. The crack increases the flexibility of micro-beam at the crack location and changes its characteristics such as mode shape, natural frequency, and dynamic response. Micro-electromechanical structures are made of brittle material, such as polycrystalline silicon (polysilicon) under potentially severe mechanical and environmental loading conditions. These structures may be subjected to high frequency and cyclic loading conditions. The studies of the failure analysis associates show that silicone materials are brittle in ambient air and there is the possibility of the initiation and growth of the crack in the silicone MEMS devices [17]. Studies in the field of faulty micro-beams show that the crack leads to a reduction in the pull-in voltage and the structure triggers pull-in bellow the predicted pull-in voltage of intact micro-beam. Motallebi et al. [18] investigated the effects of the open crack depth ratio, crack position and crack number on static and dynamic pull-in voltage in the cracked micro-beam with clamped-clamped and cantilever boundary condition. In their work, the dynamic pull-in voltage is defined as a step DC voltage, which is applied suddenly and leads to instability of the system. They solved the governing equations through SSLM and Galerkin-based reduced order model. They showed that the presence of cracks leads to reduce the static pull-in because of natural frequency reduction. Zhou et al. [19] investigated the effects of slant open crack on the mode shape, natural frequency and pull-in voltage of fixed-fixed micro-beam in different crack depths, location, and slant angle. They showed that the crack position has a more significant influence on the pull-in voltage value than the slant angle or the depth ratio. They extracted the dynamic response of fixed-fixed micro-beam in the condition far from pull-in phenomenon using single mode and perturbation method and investigated the effects of the crack on the peak amplitude and resonance frequency. According to their research, by increasing the depth of crack, the natural frequency of cracked micro-beam decreases and also by increasing the slant angle the natural frequency increases. But the effects of the crack on the natural frequency are different in different modes.

It is possible to suppress the electrostatically induced pull-in instability in the cracked micro-beam by applying a range of values of the amplitude and the frequency of the high-frequency AC voltage excitation in the post pull-in state. The aim of the present research is to provide a method based on high-frequency voltage excitation to identify the crack in the cracked micro-beam and introduce range of values of the amplitude and the frequency which leads to suppressing the pull-in phenomenon in the post pull-in state. To this end, the cracked micro-beam is modelled as a single-degree-of-freedom that consists of mass-spring-damper and the equation of motion is extracted. The stiffness of the model is calculated by static pull-in voltage. To solve the extracted equation, it is used the direct partition of motion and then the curves of averaged position of motion of cracked micro-beam are extracted versus amplitude and frequency of high-frequency excitation for different crack depths and locations in the post pull-in state. Also, a safe zone of voltage in which the cracked micro-beam vibrates stably, despite being in post pull-in state, are extracted for different crack depths and locations. These zones are drawn in the voltage amplitude-voltage frequency plane. The areas of these zones are different versus different depths and locations of crack and then one can identify the presence of a crack in the cracked micro-beam using these different areas. For this purpose, the high-frequency voltage with constant amplitude and variable frequency is applied to cracked micro-beam. The frequency is changed continuously until the pull-in phenomenon occurs. Using the parameters of voltage (excitation amplitude and frequency), in which the pull-in occurs, the crack's parameters are extracted.

2- Mathematical Modelling

A general view of single-edge cracked cantilever MEMS devices is illustrated in Fig. 1. It is used one degree of freedom model to evaluate the vibration of microstructure and the crack effects. Fig. 2 shows a schematic view of one degree of freedom model of cracked cantilever micro-beam under DC polarization voltage and an AC voltage composed of moveable mass and spring and damper. The moveable part of this microstructure is a cracked micro-beam. The governing equations of the lateral vibration of cracked micro-beam are [15, 20]:

$$m\ddot{x}+c\dot{x}+kx=\frac{\varepsilon A}{2(d-x)^2}(V_{DC}+V_{AC}\cos(\Omega t))^2 \qquad (1)$$

where x is the displacement of the cracked micro-beam, m is a mass of cracked micro-beam and c is the damping of the system, which represents energy dissipation. k is the equivalent stiffness of cracked micro-beam considering the effect of elastic force, residual axial force and crack. The dielectric constant is denoted ε and d is the initial capacitor gap width, A is area of cross section. It is assumed that there is overlapping between moving micro-beam and the stationary electrode. V_{DC} is a DC voltage, whereas V_{AC} and Ω are the amplitude and the frequency of the AC actuation. The stiffness of the cracked micro-beam is denoted [21, 22]:

$$k=\frac{27\varepsilon A V_{pull-in}^2}{8d^3} \qquad (2)$$

where $V_{pull-in}$ is the static pull-in voltage of cracked micro-beam obtained using reduced order model [18]. In the calculation of the static pull-in voltage, $n=a/h$ is the relative depth of crack where a is the height of crack in the cracked micro-beam and h is the height of micro-beam. Also, $\beta=L_0/L$ is the relative location of the crack where L_0 is the distance of crack from fixed end and L is the length of micro-beam.

In order to facilitate the analysis and understand the vibration feature of the system, the non-dimensional equation of motion is given by:

$$X''+2\xi X'+X=\frac{\alpha}{(1-X)^2}+\frac{\lambda}{(1-X)^2}\cos(\Omega_{non}\tau)+\frac{\gamma}{(1-X)^2}\cos(2\Omega_{non}\tau) \qquad (3)$$

Where $X=x/d$ and $\tau=\omega t$ the primes denote the derivatives with respect to τ. $\xi=c/2m\omega$ and $\omega=\sqrt{k/m}$ are non-dimensional damping coefficient and natural frequency of cracked micro-beam, respectively. $\Omega_{non}=\Omega/\omega$ is the non-dimensional frequency of the AC actuation. The coefficients α, λ and γ are defined as:

$$\alpha=\frac{\varepsilon A}{2m\omega^2 d^3}\left(V_{DC}^2+\frac{V_{AC}^2}{2}\right),\qquad \lambda=\frac{\varepsilon A}{m\omega^2 d^3}V_{DC}V_{AC},\qquad \gamma=\frac{\varepsilon A}{4m\omega^2 d^3}V_{AC}^2, \qquad (4)$$

The coefficients α, λ contain DC and AC excitation voltage and γ contains AC excitation voltage. These coefficients are related by the following expression:

$$\lambda^2=8\gamma(\alpha-\gamma) \qquad (5)$$

The value of the static pull-in voltage, $V_{pull,in}$, is dependent on the parameters of a crack such as depth and location of it in the cracked micro-beam. Thus, the stiffness and natural frequency of cracked micro-beam are changed by changing the parameters of crack. Therefore, the value of the each of non-dimensional parameters of Eq. (4) changes depending on depth and location of the crack.

Eq. (1) contains slow and fast dynamics of cracked micro-beam. Slow dynamics describes main motion at the time-scale of natural vibrations of cracked micro-beam and fast dynamics is at the time-scale of high-frequency voltage. The main equation governing slow dynamics of the structure is extracted using the method of the direct partition of motion [15, 23]. This method is implemented by introducing two different time-scales; fast time $T_0=\eta^{-1/2}\tau$ and slow time $T_1=\tau$ and the non-dimensional displacement of the mass $X(\tau)$ split up into a slow part $Z(T_1)$ and a fast part $\phi(T_0,T_1)$ which contain both slow and fast dynamics , respectively:

$$X(\tau)=Z(T_1)+\phi(T_0,T_1)=Z(T_1)+\eta^2\tilde{\phi}(T_0,T_1) \qquad (6)$$

Here positive parameter η is introduced to measure smallness of other parameters ($0<\eta<<1$). Therefore, $\eta^2\tilde{\phi}(T_0,T_1)$ is small compared with $Z(T_1)$ and the non-dimensional excitation frequency is denoted $\Omega_{non}=\eta^{-1/2}$.

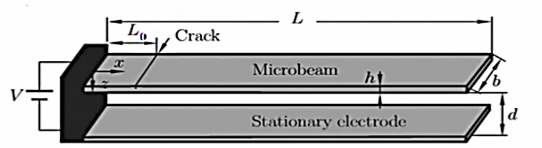

Fig. 1. A schematic view of electrically actuated cracked cantilever micro-beam

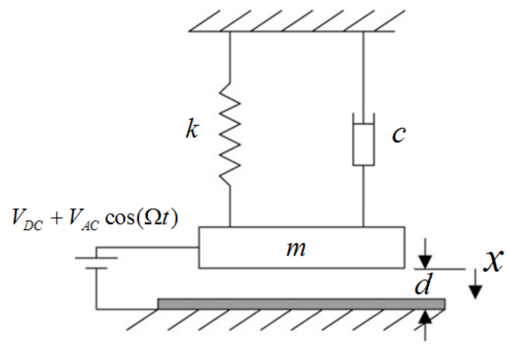

Fig. 2. A single-degree-of-freedom model of MEMS system

The fast motion and its derivatives are assumed to be 2π -periodic functions of the fast time T_0 with zero mean value with respect to it and the fast time averaging operator is denoted $\langle.\rangle=\frac{1}{2\pi}\int_0^{2\pi}(.)dT_0$, where $\langle X(\tau)\rangle=Z(T_1)$. Introducing $D_m^n=\frac{\partial^n}{\partial T_m^n}$, yields:

$$\frac{d}{d\tau}=\eta^{-1/2}D_0+D_1 \qquad (7)$$

$$\frac{d^2}{d\tau^2}=\eta^{-1}D_0^2+\eta^{-1/2}2D_0D_1+D_1^2 \qquad (8)$$

Substituting Eqs. (7) and (8) into Eq. (3) yields the following equation:

$$\eta(D_0^2\tilde{\phi})+2\eta^{3/2}(D_0D_1\tilde{\phi})+\eta^2(D_1^2\tilde{\phi})+(D_1^2Z)+ 2\xi[\eta^{3/2}(D_0\tilde{\phi})+\eta^2(D_1\tilde{\phi})+(D_1Z)]+Z+\eta^2\tilde{\phi}= \frac{\alpha}{(1-Z-\eta^2\tilde{\phi})^2}+\frac{\lambda}{(1-Z-\eta^2\tilde{\phi})^2}\cos(T_0)+ \frac{\gamma}{(1-Z-\eta^2\tilde{\phi})^2}\cos(2T_0) \quad (9)$$

The non-dimensional parameters of Eq. (4) are denoted $\lambda=\eta\tilde{\lambda}$, $\gamma=\eta\tilde{\gamma}$ and $\varepsilon=\eta\tilde{\varepsilon}$. The dominant terms depending on T_0 up to the order $O(\eta)$ in Eq. (9) are:

$$(D_0^2\tilde{\phi})=\frac{\tilde{\lambda}}{(1-Z)^2}\cos(T_0)+\frac{\tilde{\gamma}}{(1-Z)^2}\cos(2T_0) \quad (10)$$

Therefore, the response of fast motion of cracked micro-electromechanical system is given by:

$$\tilde{\phi}(T_0,T_1)=-\frac{\tilde{\lambda}}{(1-Z)^2}\cos(T_0)-\frac{\tilde{\gamma}}{4(1-Z)^2}\cos(2T_0)+O(\eta) \quad (11)$$

Now, averaging Eq. (9) over a period of the fast time scale T_0 leads to the following equation:

$$(D_1^2Z)+2\xi(D_1Z)+Z=\left\langle\frac{\alpha}{(1-Z-\eta^2\tilde{\phi})^2}\right\rangle+ \left\langle\frac{\lambda}{(1-Z-\eta^2\tilde{\phi})^2}\cos(T_0)\right\rangle+\left\langle\frac{\gamma}{(1-Z-\eta^2\tilde{\phi})^2}\cos(2T_0)\right\rangle \quad (12)$$

where up to order $O(\eta^3)$:

$$\left\langle\frac{\alpha}{(1-Z-\eta^2\tilde{\phi})^2}\right\rangle=\frac{\alpha}{(1-Z)^2} \quad (13)$$

$$\left\langle\frac{\lambda}{(1-Z-\eta^2\tilde{\phi})^2}\cos(T_0)\right\rangle=-\frac{\tilde{\lambda}^2}{(1-Z)^5} \quad (14)$$

$$\left\langle\frac{\gamma}{(1-Z-\eta^2\tilde{\phi})^2}\cos(2T_0)\right\rangle=-\frac{\tilde{\gamma}^2}{4(1-Z)^5} \quad (15)$$

Consequently, the equation of the slow dynamics obtained from Eq. (12) can be written as:

$$(D_1^2Z)+2\xi(D_1Z)+Z=\frac{\alpha}{(1-Z)^2}-\frac{4\lambda^2+\gamma^2}{4\Omega_{non}^2(1-Z)^5} \quad (16)$$

Eq. (16) is the governing equation of slow dynamics of cracked micro-beam and the last term of the right-hand side of this equation represents the effect of high-frequency on slow motion of the mass. The zeros of the slow dynamics Eq. (16) are the averaged positions around which the periodic solutions of Eq. (1) oscillate. To obtain these fixed points, the following equation should be solved as:

$$Z(1-Z)^5=\alpha(1-Z)^3-\frac{4\lambda^2+\gamma^2}{4\Omega_{non}^2} \quad (17)$$

Finally, the solution of Eq. (6) up to order $O(\eta^3)$ is extracted as:

$$X(\tau)=Z-\frac{\lambda}{\Omega_{non}^2(1-Z)^2}\cos(\Omega_{non}\tau)- \frac{\lambda}{4\Omega_{non}^2(1-Z)^2}\cos(2\Omega_{non}\tau)+O(\eta^3) \quad (18)$$

When the cracked micro-beam is under only the DC excitation and in the absence of high-frequency AC excitation ($\lambda=0,\gamma=0$), the steady state displacement is obtained i.e., $Z(1-Z)^2=\alpha$. In this case, if the DC excitation voltage equal static pull-in voltage, using the combination of Eqs. (2) and (4), the non-dimensional parameter is $\alpha_p=\frac{\varepsilon A}{2m\omega^2d^3}(\frac{8kd^3}{27\varepsilon A})=0.1418$

Thus, the steady state static displacement is obtained $Z_p=1/3$. Even though the stiffness and natural frequency and then corresponding static pull-in voltage of cracked micro-beam change for different crack depths and locations, the critical parameters $\alpha_p=0.1481$ and $Z_p=1/3$ are the same for intact and cracked micro-beam. These parameters are independent of the dimension of micro-beam and severity of crack.

If the magnitude of the non-dimensional parameter is $\alpha>0.1481$, the pull-in phenomenon occurs and leads to an overlap between the cracked micro-beam and stationary electrode and causes destruction and disability of MEMS device. According to the research on intact micro-beam [15], it is possible to suppress the pull-in phenomenon using a high-frequency excitation voltage. In this case, the electrostatically induced pull-in instability is suppressed for a range of values of the amplitude and the frequency of the high-frequency AC. This range was shown as a curve on the plane (Ω_{non},γ) [15].

In this research, by applying high-frequency AC excitation voltage on cracked micro-beam, the stable zones of vibration are extracted for different crack severities in the post pull-in state. The effects of crack on these zones are investigated, as well

3- Effect of Crack on Averaged Position of Vibration in Cracked Micro-beam

By applying the voltage excitation on the cracked micro-beam for a range of the amplitude and the frequency of high-frequency AC, the electrostatically induced pull-in instability is suppressed. Thus, the cracked micro-beam vibrates around special position named the averaged position of vibration.

In this section, the effects of depth and location of crack are investigated on the averaged position of vibration of cracked micro-beam for a range of amplitude and frequency of the high-frequency AC excitation in the post pull-in state. For this purpose, a cracked cantilever micro-beam with properties listed in Table 1 and initial capacitor gap width d=1.18μm is considered under axial residual force N_r=0.0003N. Figs. 3 to 7 show the stationary solutions Z of the slow dynamics Eq. (17) versus constant DC voltage V_{DC}=5.7 V and amplitude of AC voltage V_{AC}=9.88 V and different frequencies for different crack depths and locations. For constant excitation frequency, two averaged position of vibration for provided a model of cracked micro-beam are extracted, one of them is stable position and the other is the unstable position.

In Figs. 3 to 7, the lower branch (solid lines) is the stable averaged position and the upper branch (dashed line) is the unstable averaged position. For the constant excitation amplitude and frequency, by increasing the relative depth of crack or approaching the crack to the fixed end, the value of the averaged position of vibration increases in stable state and decreases in an unstable state.

As can be seen in Figs. 3 to 7, in lower frequency, the averaged position of vibration are closer together versus different crack locations and by increasing the frequency, these positions separate from each other.

Table 1. The value of design variables in cracked cantilever micro-beam [18]

Design Variable	Scientific Symbol	value
Length	L	250 μm
Width	b	50 μm
Height	h	3 μm
Density	ρ	2331 kg/m^3
Young's Modulus	E	169 GPa
Poisson's Ratio	ν	0.06
Dielectric Constant	ε	8.85 pF/m

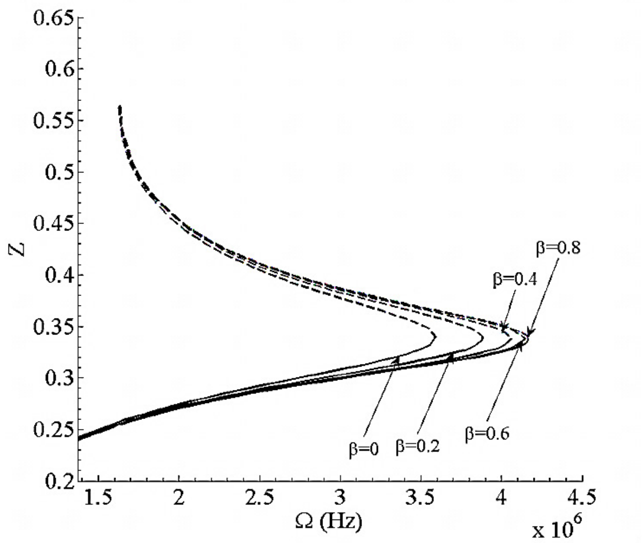

Fig. 3. Averaged position of vibration versus excitation frequency for V_{DC}=5.7 V, V_{AC}=9.88 V and relative depth ratio n=0.1 and different relative crack locations β

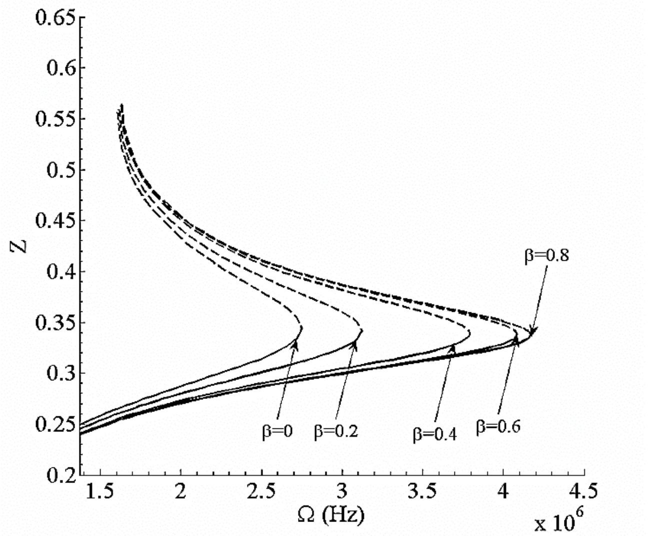

Fig. 4. Averaged position of vibration versus excitation frequency for V_{DC}=5.7 V, V_{AC}=9.88 V and relative depth ratio n=0.2 and different relative crack locations β

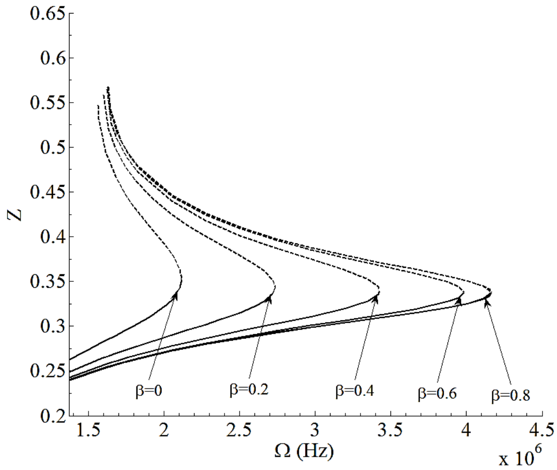

Fig. 5. Averaged position of vibration versus excitation frequency for V_{DC}=5.7 V, V_{AC}=9.88 V and relative depth ratio n=0.3 and different relative crack locations β

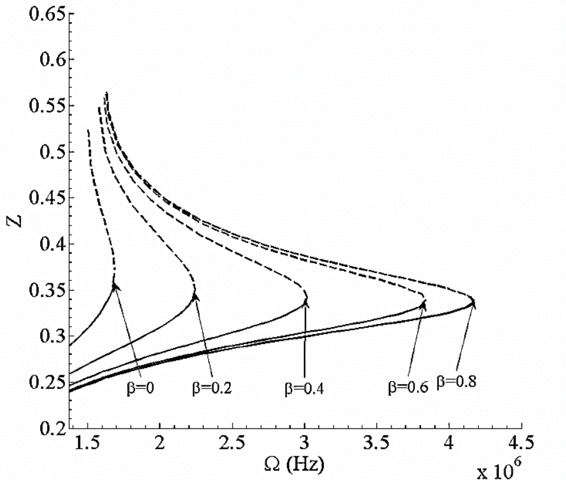

Fig. 6. Averaged position of vibration versus excitation frequency for V_{DC}=5.7 V, V_{AC}=9.88 V and relative depth ratio n=0.4 and different relative crack locations β

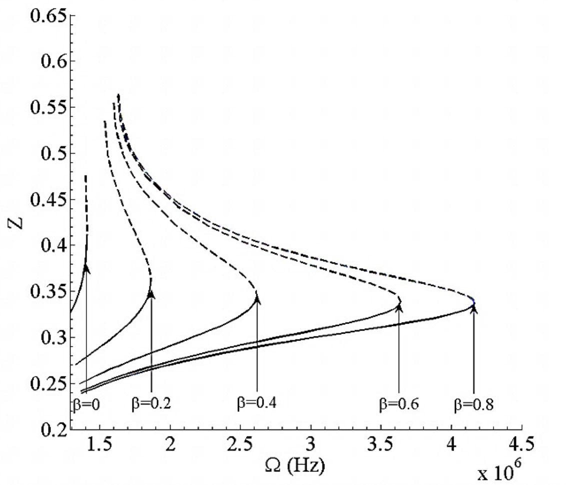

Fig. 7. Averaged position of vibration versus excitation frequency for V_{DC}=5.7 V, V_{AC}=9.88 V and relative depth ratio n=0.5 and different relative crack locations β

In these figures, by increasing the depth of crack or approaching crack to the fixed end, it is possible to suppress the pull-in instability in the lower range and then the possibility of instability increases. It follows from the comparison of figures that the location of crack has more influence on the averaged position of vibration than crack depth.

Figs. 8 to 12 show the stationary solutions Z of the slow dynamics Eq. (17) versus constant frequency of AC voltage Ω=272kHz and different voltage amplitudes for different crack depths and locations. In these figures, two averaged position of vibration for provided model of cracked micro-beam are extracted versus constant amplitude of excitation voltage. The solid lines represent the stable averaged position and the dashed line are the unstable averaged position.

As can be seen in curves of Figs. 8 to 12, by increasing the depth of crack or approaching the crack to the fixed end, the area of stable zone decrease and then the suppression of pull-in instability is possible in a lower range of AC voltage. Also, by increasing the depth of crack or approaching the crack to the fixed end, the stable averaged position of vibration gets closer to unstable averaged position of vibration.

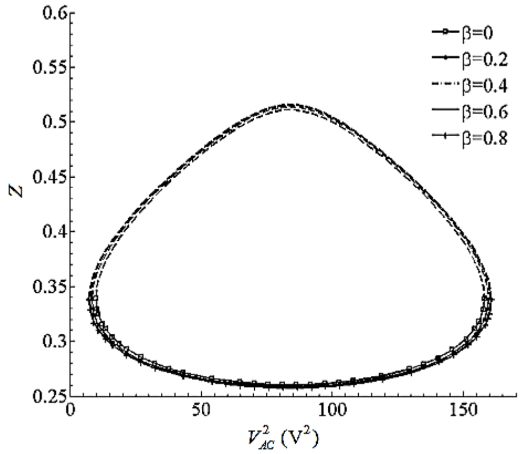

Fig. 8. Averaged position of vibration versus square of AC voltage amplitude for Ω=272 kHz and relative crack depth n=0.1 and different relative crack locations β

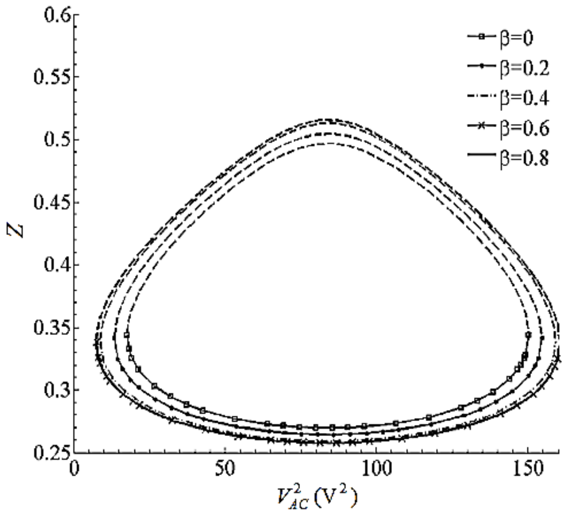

Fig. 9. Averaged position of vibration versus square of AC voltage amplitude for Ω=272 kHz and relative crack depth n=0.2 and different relative crack locations β

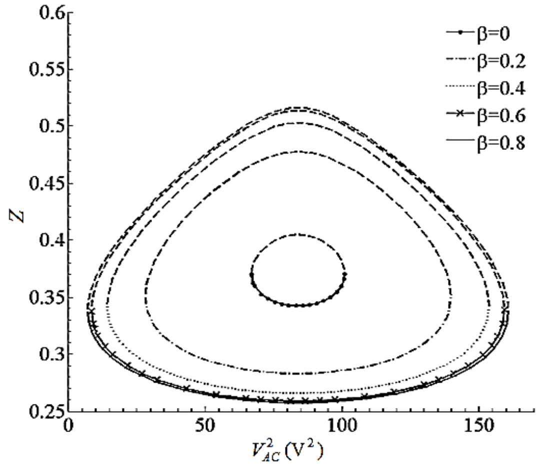

Fig. 10. Averaged position of vibration versus square of AC voltage amplitude for Ω=272 kHz and relative crack depth n=0.3 and different relative crack locations β

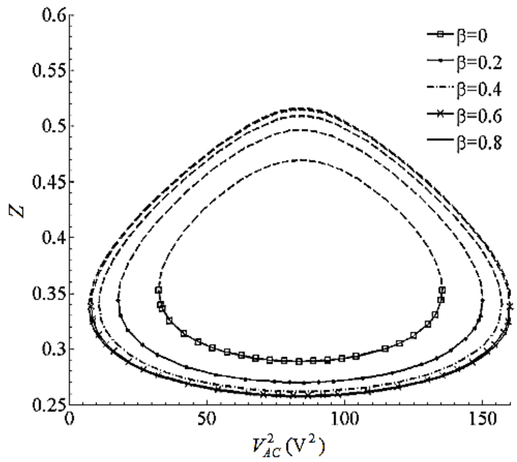

Fig. 11. Averaged position of vibration versus square of AC voltage amplitude for Ω=272 kHz and relative crack depth n=0.4 and different relative crack locations β

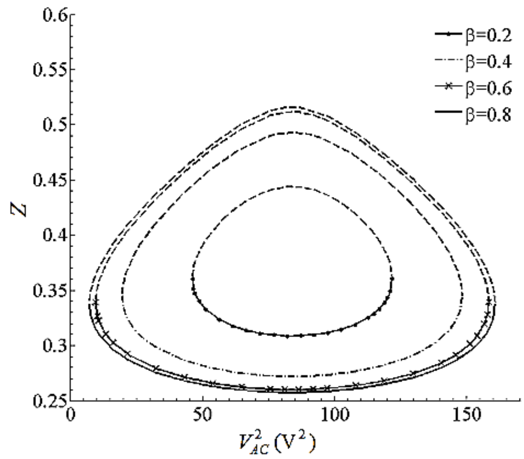

Fig. 12. Averaged position of vibration versus square of AC voltage amplitude for Ω=272 kHz and relative crack depth n=0.5 and different relative crack locations β

By comparing Figs. 3 to 12, it becomes clear that the zones, created between stable and unstable branches, are more sensitive to the location of crack than to depth. Thus, the location of crack has more influence on stable averaged position of vibration than the crack depth.

4- Effects of Crack on Range of Amplitude and Frequency of High-Frequency AC Excitation in Stable Zone

Similar to intact micro-beam, by applying high-frequency AC excitation, it is possible to suppress pull-in instability in cracked micro-beam in the post pull-in state. In this section, the effect of high-frequency AC excitation on the suppression of pull-in instability in cracked micro-beam is investigated in post pull-in state. Then, the stable zones of vibration of MEMS are extracted versus amplitude and frequency of AC excitation in the plane (Ω, V_{AC}).

Fig. 13 shows the time history of cracked micro-beam with properties cited in Table 1 and relative crack depth n=0.3 and relative crack location β=0.4 under DC excitation in the post pull-in state, α=0.1509, λ=γ=0. As can be seen in this figure, the pull-in phenomenon occurs. This instability is characterized by the slope of the displacement approaching infinity.

Fig. 14 shows the time history of the same cracked micro-beam under DC and high-frequency AC excitation, α=0.1509, γ=0.07 and Ω_{non}=3.1. As regards, α>0.1481, the cracked micro-beam is in post pull-in state. As can be seen, there is no trace of the pull-in phenomenon. By comparing Figs. 13 with 14, it can be concluded that by applying the high-frequency AC voltage excitation, the pull-in phenomenon is suppressed in the cracked micro-beam and then the system vibrates stably.

Figs. 15 to 19 show the pull-in suppression zones of cracked micro-beam versus different depths and locations of crack in the post pull-in state. These curves are extracted for cantilever cracked micro-beam with properties cited in Table 1 and under axial residual force N_r=0.0003N and versus DC and AC excitation voltage $V^2_{DC}+V^2_{AC}/2$=81.33 .

These zones show the range of amplitude and frequency of high-frequency voltage excitation leading to the stable vibration of cracked micro-beam in the post pull-in state, α >0.1481. In order to extract the zone of stable vibration in the post pull-in state, it is derived from Eq. (17) respect to Z and then the critical value of the stable position of vibration of cracked micro-beam is extracted. By substituting extracted critical values to Eq. (17), the curves of stable zones are extracted.

In Figs. 15 to 19, in the lower frequencies, it is possible to suppress the pull-in phenomenon in a wide range of excitation voltage. By increasing the frequency of AC voltage excitation, the range of voltage, which leads the system to vibrate stably,reduces. By comparing figures, it can be seen that for the constant relative depth of crack, by approaching the crack to the fixed end, the stable zone becomes smaller and the possibility of pull-in phenomenon occurrence increases. Also, by increasing the depth of crack, the stable zone becomes smaller.

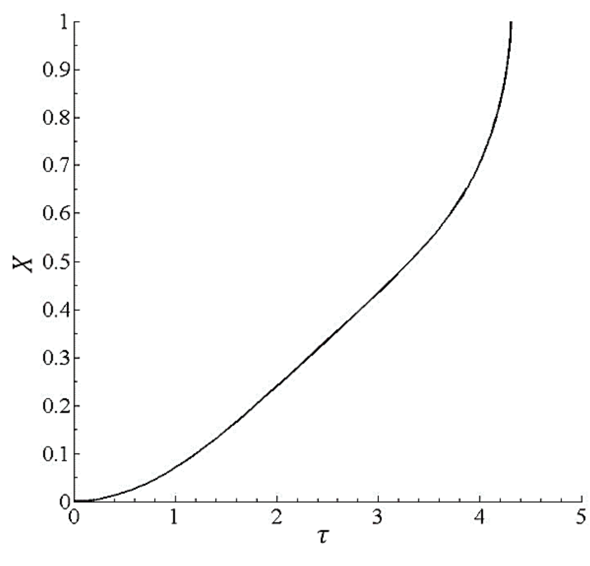

Fig. 13. Time history of Eq. (3) for cracked micro-beam with relative crack depth n=0.3 and relative crack location β=0.4 for α=0.1509, λ=γ=0

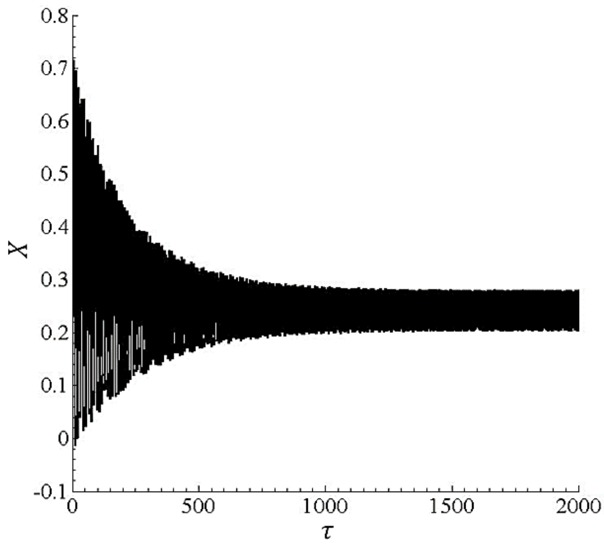

Fig. 14. Time history of Eq. (18) for cracked micro-beam with relative crack depth n=0.3 and relative crack location β=0.4 for α=0.1509, γ=0.07 and Ω_{non}=3.1

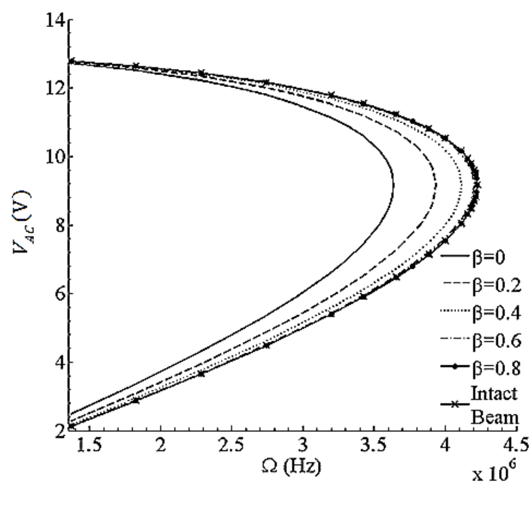

Fig. 15. Suppression zone of the pull-in instability versus $V^2_{DC}+V^2_{AC}/2$=81.33 and relative depth ratio n=0.1 and different relative crack locations β

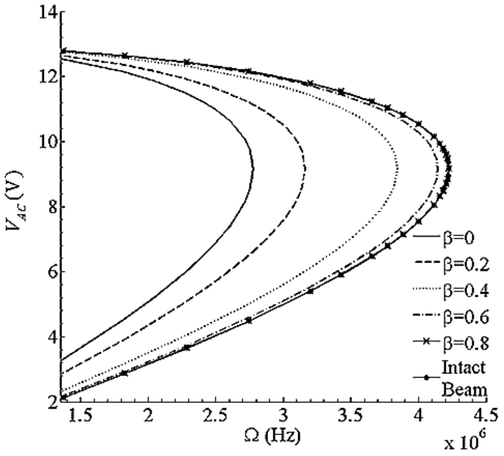

Fig. 16. Suppression zone of the pull-in instability versus $V^2_{DC}+V^2_{AC}/2$=81.33 and relative depth ratio *n*=0.2 and different relative crack locations *β*

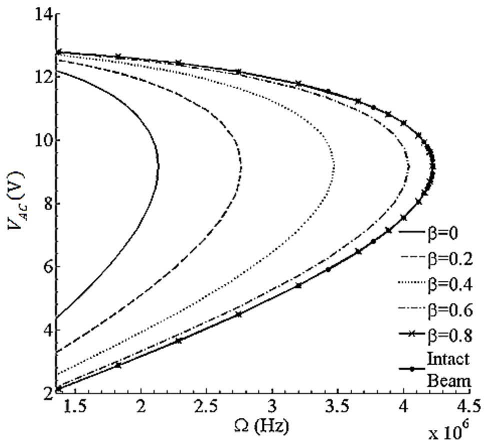

Fig. 17. Suppression zone of the pull-in instability versus $V^2_{DC}+V^2_{AC}/2$=81.33 and relative depth ratio *n*=0.3 and different relative crack locations *β*

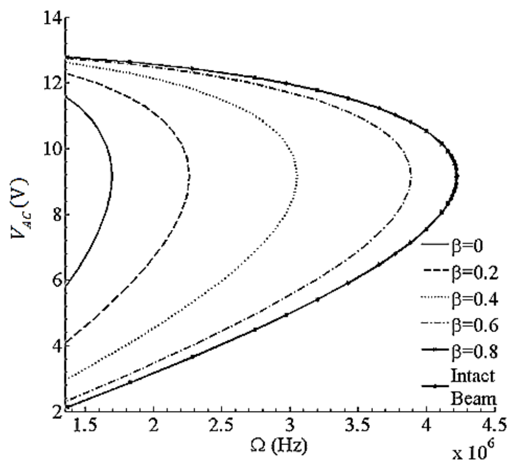

Fig. 18. Suppression zone of the pull-in instability versus $V^2_{DC}+V^2_{AC}/2$=81.33 and relative depth ratio *n*=0.4 and different relative crack locations *β*

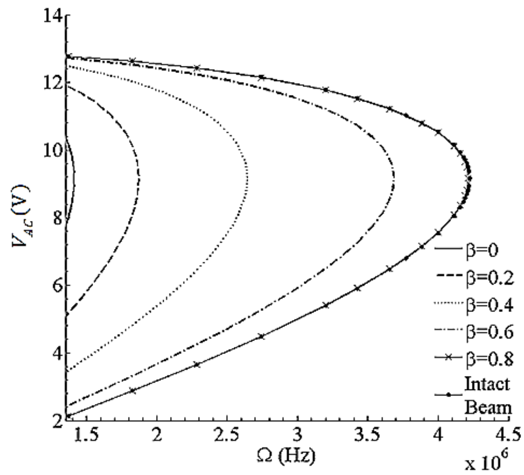

Fig. 19. Suppression zone of the pull-in instability versus $V^2_{DC}+V^2_{AC}/2$=81.33 and relative depth ratio *n*=0.5 and different relative crack locations *β*

As can be seen in Figs. 15 to 19, the crack has more influence on the stable zone and the size of stable zone changes versus the severity of crack. One can use this effect and the change in size of areas to detect the crack in the micro-beam.

In order to verify proposed model and solution and extracted curves, Figs. 20 to 22 are extracted using a direct partition of motion and Runge–Kutta method. The value of non-dimensional damping factor is ζ=0.01. These figures show a good agreement between analytical and numerical solution.

5- Conclusion

Crack is one of the main and most common faults in the MEMS, if not diagnosed, leads to early failure and disability in MEMS devices. The presence of crack in the system causes the changes in the characteristics of system and one can detect the crack and its severity using these changes and a strong indicator. One of the crack detection methods is applying the high-frequency voltage excitation on MEMS devices in the post pull-in state and investigating effects of crack on the cracked micro-beam in the presence of this excitation.

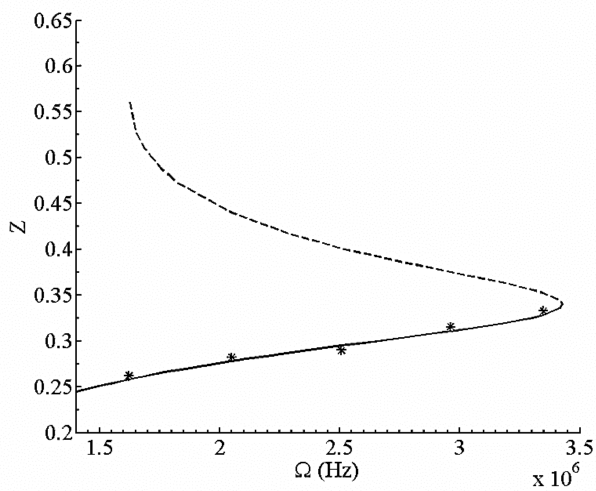

Fig. 20. Averaged position of vibration versus excitation frequency for V_{DC}=5.7 V, V_{AC}=9.88 V, *n*=0.3 and *β*=0.4: using direct partition of motion method (-) and Runge-Kutta method (*)

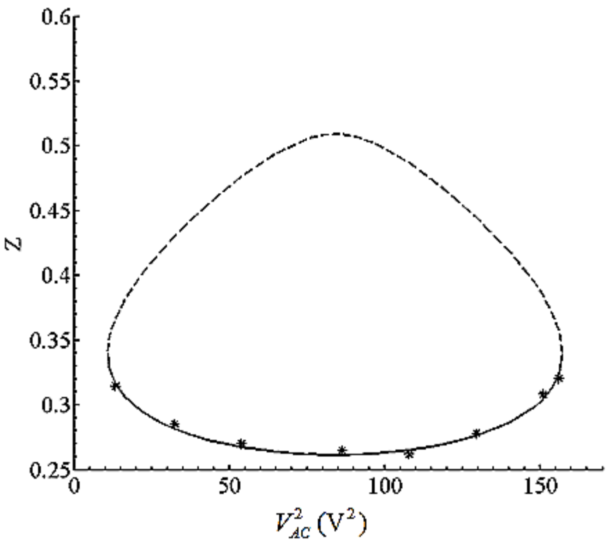

Fig. 21. Averaged position of vibration versus square of AC voltage amplitude for *Ω*=272 kHz, *n*=0.3 and *β*=0.4: using direct partition of motion method (-) and Runge-Kutta method (*)

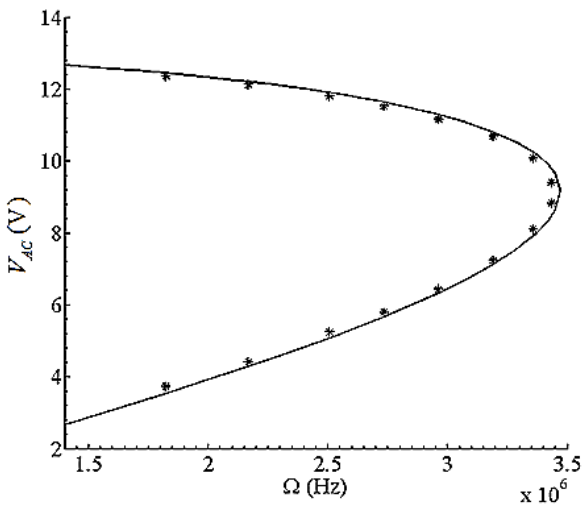

Fig. 22. Suppression zone of the pull-in instability versus $V^2_{DC}+V^2_{AC}/2$=81.33 and for *n*=0.3 and *β*=0.4: using direct partition of motion method (-) and Runge-Kutta method (*)

It is possible to suppress the electrostatically induced pull-in instability in the cracked micro-beam for the range of values of the amplitude and the frequency of the high-frequency AC voltage excitation in the post pull-in state. In this study, the effects of the parameters of a crack such as depth and location on this stable range were investigated.

The cracked micro-beam was modelled as a single-degree-of-freedom system consisting of mass-spring-damper and the effects of crack were assumed in the stiffness of spring. The stiffness of mentioned model was extracted using static pull-in voltage. The extracted equation was solved using a direct partition of motion and the main equation governing slow dynamics of the structure was solved using averaging technique. By the solution of slow dynamics, the effect of crack on the averaged position of vibration of cracked micro-beam was investigated versus the amplitude and the frequency of the high-frequency AC, in the post pull-in state. Also, the influence of a high-frequency actuation on the suppression of the pull-in instability in the cracked micro-beam was investigated versus crack severity.

The zone of suppression of pull-in instability was extracted in the plane (Ω, V_{AC}).By approaching the crack to the fixed end or increasing depth of a crack in the cracked micro-beam, the stable zone becomes smaller and it is possible to suppress the pull-in phenomenon in a smaller range of amplitude and frequency of AC excitation. The boundary of stable zones are separated from each other versus different depths and locations of crack and one can detect the crack in MEMS devices using these different zones. These zones are a strong index in troubleshooting of cracked micro-beam.

This method can be used in sensors' health-monitoring in the future work. For this purpose and prediction the crack in the micro-beam, the cracked micro-beam can be excited under DC voltage and AC high-frequency actuation with constant amplitude and different frequencies. By continuously changing frequency, the pull-in phenomenon occurs and therefore the value of frequency and amplitude of pull-in instability is extracted. One can predict parameters (depth and location) of the crack using the value of amplitude and frequency of excitation in the pull-in state and an optimization method such as a genetic algorithm.

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