

Three-Dimensional Optimization of Blade Lean and Sweep for a Transonic Axial Compressor and Investigation of the On-Design and Off-Design Engine Performance

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ABSTRACT: Recently, optimization methods have been considered by authors to enhance the turbo-machines' performance. In this article, the genetic algorithm (GA) and artificial neural network (ANN) with computational fluid dynamics (CFD) are being coupled, and the optimization of NASA Rotor-67, an axial compressor, has been simulated. The compressor flow field is simulated with CFD, and the results proved the excellent validation with experimental data. The rotor leaned and swept parametrization was modeled, and the results are improvements in design objective functions: pressure ratio, isentropic efficiency, and mass flow rate. According to the best-optimized case results, the mass flow rate, pressure ratio, and isentropic efficiency of the design point have been increased by about 2.020%, 1.297%, and 0.174%, respectively. Improving the convergence of surface streamlines in delaying the shock on the blade is another factor in improving the optimal rotor's performance compared to the base one. Then, the effect of the best-optimized rotor is studied at the on-design and off-design steady-state performance of a turbojet engine. The matching code has been worked out by solving compatibility equations using the characteristic maps. The results show that Thrust has improved at design and off-design speeds.

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1- Introduction

The performance enhancement of axial compressors, which are widely used in industries, has been a major challenge for engineers, requiring an interactive approach to parameters such as pressure ratio and performance efficiency. Three-dimensional geometric changes in compressor blades have been the primary means to achieve the desired results, achieved through targeted changes in lean and swept blade. Examples of geometric changes in leaned and swept blades in compressors are mentioned below, where artificial intelligence coupled with computational fluid dynamics has yielded results. The authors have also studied some research in the numerical optimization and design of experiment (DOE) field of axial compressors, which has led to improvement in the surge margin and near-stall isentropic efficiency of axial [1] and radial compressors [2].

A study on optimizing the leaned and swept blade of an axial compressor stage was conducted by Zhongyi et al, where optimal blade geometry was found through numerical optimization, resulting in the extraction of three near-stall design points and close-to-stall performance improvement, including improved isentropic efficiency and significant surge margin increase [3].

Research on geometric changes and achieving new performance from the LPC module of a turbofan engine was

carried out by Eggers et al, considering the high-pressure distortion effects coming from the fan on its booster. Leaned and swept blades were also used to change the geometry of the booster inlet guide vanes, resulting in a pressure drop improvement of up to 18% [4].

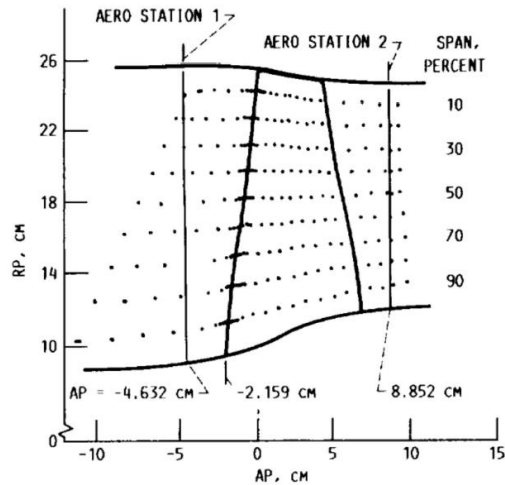
In a study published by Hamaguchi et al, the effects of forward sweep on increasing the stability margin of a centrifugal compressor were evaluated, where the upstream flow distortion affects it. According to the extracted results, forward sweep effects have a significant impact on the surge margin and rotor stability [5].

The geometric modification of a swept impeller in the design point and off-design conditions has been studied by Li et al [6]. The modified sweep geometry includes a lower loss and pressure drop compared to the baseline geometry due to the displacement of shock formation on the impeller surface.

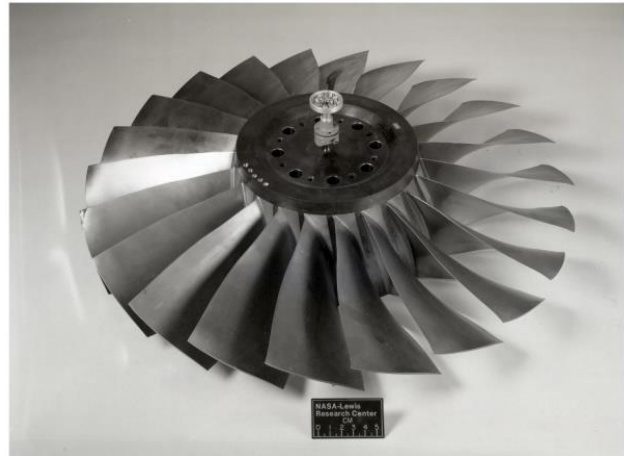
Benini studied NASA Rotor-37 geometry by genetic algorithm to aerodynamically optimize the blade stacking line [7, 8]. This paper aims to achieve a maximum of two target parameters, pressure ratio and isentropic efficiency at the design point, using lean and sweep concepts. TASKFLOW software has been used for numerical simulation. Results indicate a 5.5% increase in pressure ratio and only 0.08% decrease in isentropic efficiency.

The lean, sweep, and end-bend effects on the aerodynamic performance of a transonic compressor (NASA Rotor-37)

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(a)



(b)

Fig. 1. NASA Rotor-67 at (a) meridional and (b) three-dimensional views [13]

have been evaluated by Wang et al [9] by combining the numerical coupling method with the Shapley Additive Explanations (SHAP). The near-tip swept change has the most effect on efficiency.

The authors have previously conducted numerical research on three-dimensional optimization of axial fan and compressor using the lean and sweep methods [10, 11]. The numerical study of AGARD wing has been done similarly by authors with these process [12]. In the axial compressor studies mentioned, a 3rd order polynomial equation was employed to parametrization the three-dimensional stacking line of the axial rotor. The distinguishing feature and innovation presented in this paper is firstly, the use of spline curves to represent significant changes in the rotor stacking-line, aimed at achieving lean and sweep variations to reach higher performance points than before. These steps are elaborated upon in the continuation of the research, forming its first part. Additionally, the second part of the paper includes an analysis of the thermodynamic cycle under off-design conditions. In this context, the performance results of the optimized compressor are examined within the engine cycle, derived from a developed in-house code. The use of various objective functions in the three-dimensional optimization loop of the rotor has enabled the attainment of points with increased mass flow rates while maintaining a constant design isentropic efficiency. This enhancement allows for greater performance generation in both on-design and off-design operating conditions within the thermodynamic cycle.

So in this study, it has been attempted to achieve superior rotors by simultaneously optimizing the blade sweep and lean on an axial rotor blade, which has previously been validated using numerical tools, to increase mass flow rate and pressure

ratio. Therefore, the generated initial database has been sensitized and, with the help of neural network tools, the computational costs resulting from searching for the optimal point have been reduced, leading to the genetic algorithm. The use of geometric constraints for generating reliable geometries has significantly influenced and improved the marginal efficiency of these geometries. The results demonstrate the performance improvement of all rotor functions at the design and off-design points. Furthermore, due to investigating the effect of improving a component in the engine, the thermodynamic cycle performance (compatibility equations for steady-state conditions) has been obtained through the convergence of its compatibility equations. The results of this analysis compare the engine thrust force with the default rotor and optimized rotors, showing an improvement in engine thrust force.

2- CFD simulation of NASA Rotor-67

2- 1- Test-case

CFD analysis of a NASA axial rotor test case has been selected to study and modify the geometry for performance improvement. This rotor is located in the first stage of an axial compressor (as shown in Fig. 1), and the aerodynamic characteristics at its design point are mapped in Table 1. The available experimental results for this rotor are also data-mapped at two measurement locations used for validation with numerical data [13].

Therefore, a structured three-dimensional mesh was used for analysis and characteristic curve extraction, as shown in Fig. 2. This mesh was employed for the aerodynamic field around a single-blade to reduce computational costs. Periodic boundary conditions around the walls have been used to aid in this process.

Table 1. NASA Rotor-67 specification

Geometry specification	Number of blades	22
	Inlet tip diameter [cm]	51.4
	Exit tip diameter [cm]	48.5
	Inlet hub-to-tip radius ratio	0.375
	Exit hub-to-tip radius ratio	0.478
	Hub solidity	3.11
	Tip solidity	1.29
	Rotor aspect ratio	1.56
Performance values	Rotational speed [rpm]	16043
	Total pressure ratio	1.632
	Mass flow rate [kg/s]	33.794
	Adiabatic efficiency	0.919
	Inlet tip relative Mach number	1.38
	Tip speed [m/s]	429

2- 2- Grid study

The grid study of characteristic data from the number of computational field elements, the grid count has been increased (480,000, 720,000, 1,360,000, and 1,780,000

elements), and the variations in shapes have been depicted in the Fig. 3.

The fine grid (Grid No.3), which involves subtle changes in performance results in an acceptable range for the Yplus number (as shown in Fig. 4) has been used in generating performance curves and validation.

The distribution of the Y-plus number along the blade length indicates that, although the local Y-plus values increase in the regions near the leading and trailing edges, the overall average Y-plus value along the blade length is approximately one, according to the studies conducted by the authors of this research. Therefore, this value is considered acceptable for other research analyses.

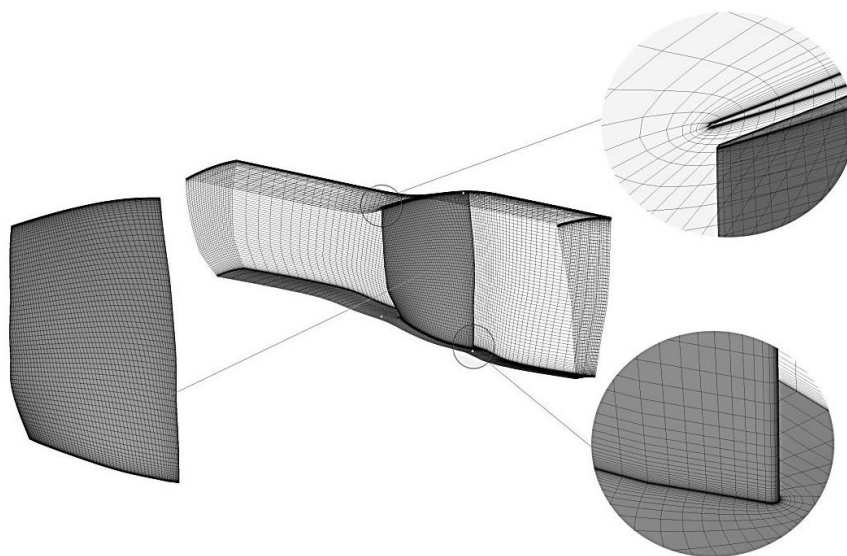
2- 3- Validation

The three-dimensional computational fluid dynamics (CFD) simulation of the specified compressor domain has been defined as illustrated in Fig. 5. To reduce computational costs, a single blade was utilized, incorporating both the inlet and outlet domains at its beginning and end.

Air enters the compressor under atmospheric conditions, and by varying the static pressure at the compressor's exit, the characteristic curve is extracted. The equations targeted in the analysis include momentum equations in three coordinate directions, the continuity equation, the energy equation, turbulence equations, and the ideal gas equation.

The numerical analysis employed the Shear Stress Transport (SST) turbulence model, with RANS equations converged to a precision of $1e-6$.

In near the compressor stall regions, where flow instability is more pronounced than in choked areas, the condition for complete convergence may not be satisfied. Therefore, the

**Fig. 2. Grids for NASA Rotor-67**

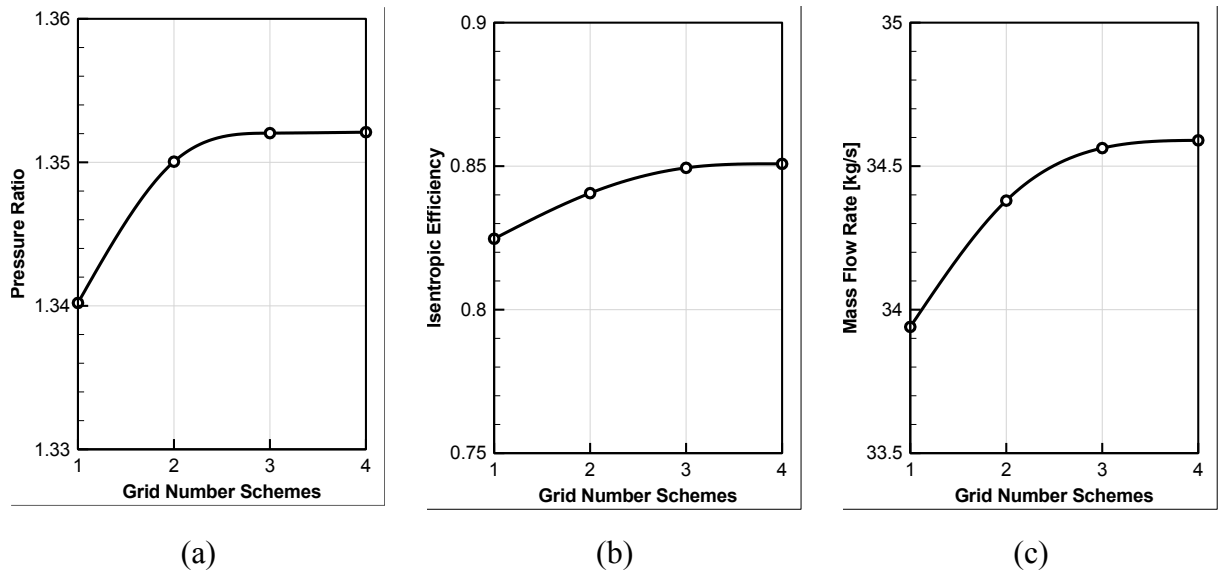


Fig. 3. Grid study of CFD results for (a) pressure ratio, (b) isentropic efficiency, and (c) mass flow rate

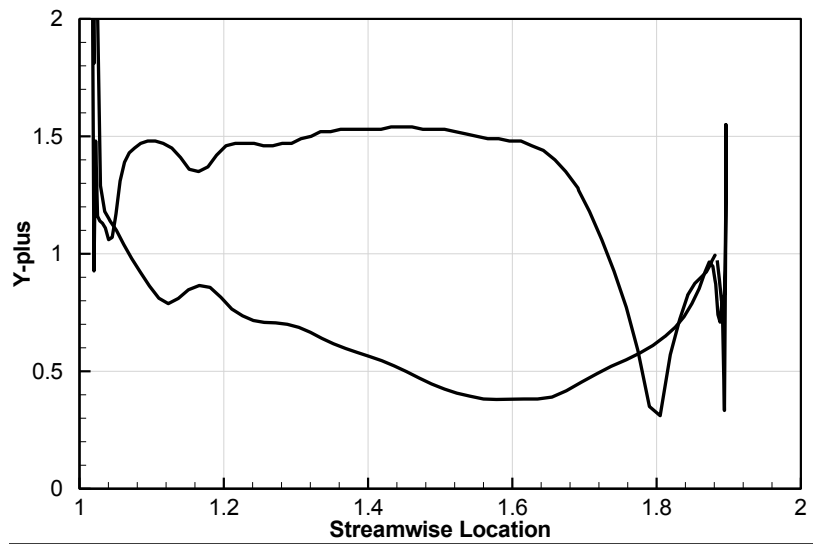


Fig. 4. NASA Rotor-67 Yplus distribution

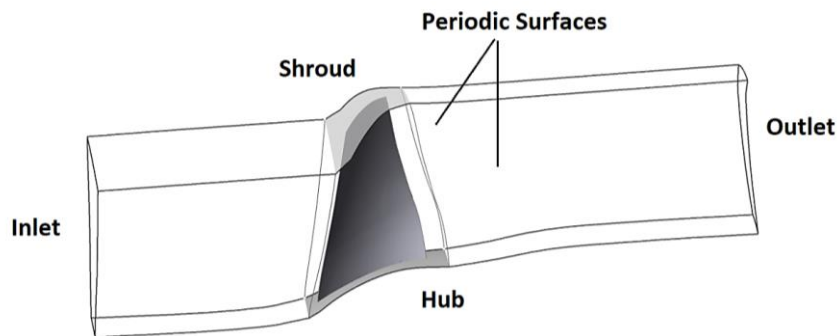


Fig. 5. Boundary conditions of Rotor-67 CFD simulation

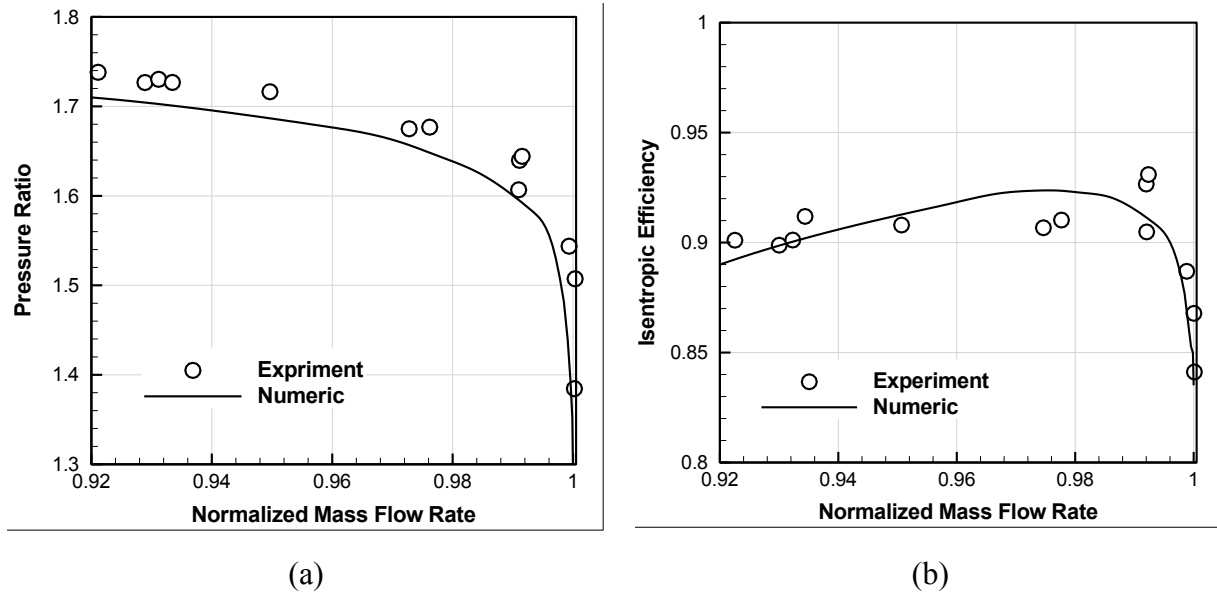


Fig. 6. Map Validation for (a) pressure ratio-mass flow rate, and (b) isentropic efficiency-mass flow rate

Table 2. Comparison of numerical results with experimental data at the design point

	Performance Parameters		
	Pressure Ratio	Isentropic Efficiency	Mass Flow Rate [kg/s]
Numerical	1.635	0.923	33.91
Experimental	1.632	0.919	33.79
Difference [%]	0.18 %	0.39 %	0.34 %

output pressure ratio was dynamically plotted to determine the cessation and convergence of the problem by stabilizing that parameter. The maximum number of convergence iterations was set to 500.

The characteristic curve extracted at the design point using numerical methods has been compared with the reference experimental results and is depicted in Fig. 6.

The comparison of performance values at the design point for the numerical method is presented in Table 2. According to the obtained results, an acceptable level of accuracy has been achieved for the numerical results.

Furthermore, the distribution of pressure ratio and isentropic efficiency of the rotor at this design point has been extracted and compared with experimental test results in Fig. 7. The very good agreement results indicate the validation of numerical results. Therefore, the numerical tool used in this

study can be an effective and efficient tool for predicting flow for optimal geometries.

3- Optimization of axial compressor

3- 1- Design variables

The compressor parametrization has changed using lean and sweep as depicted in Fig. 8. The axial-displacement of blade sections have defined as sweep [10], while the circumferential-direction of blade sections' movement has been defined as lean [10].

The spline algorithm has been used to parameterize new blade geometry. Five points were considered to design variables for each lean and sweep method. These five points each have upper and lower bounds that, by passing the curve through them, the stack line is generated.

To eliminate the stress concentration at the blade root, the

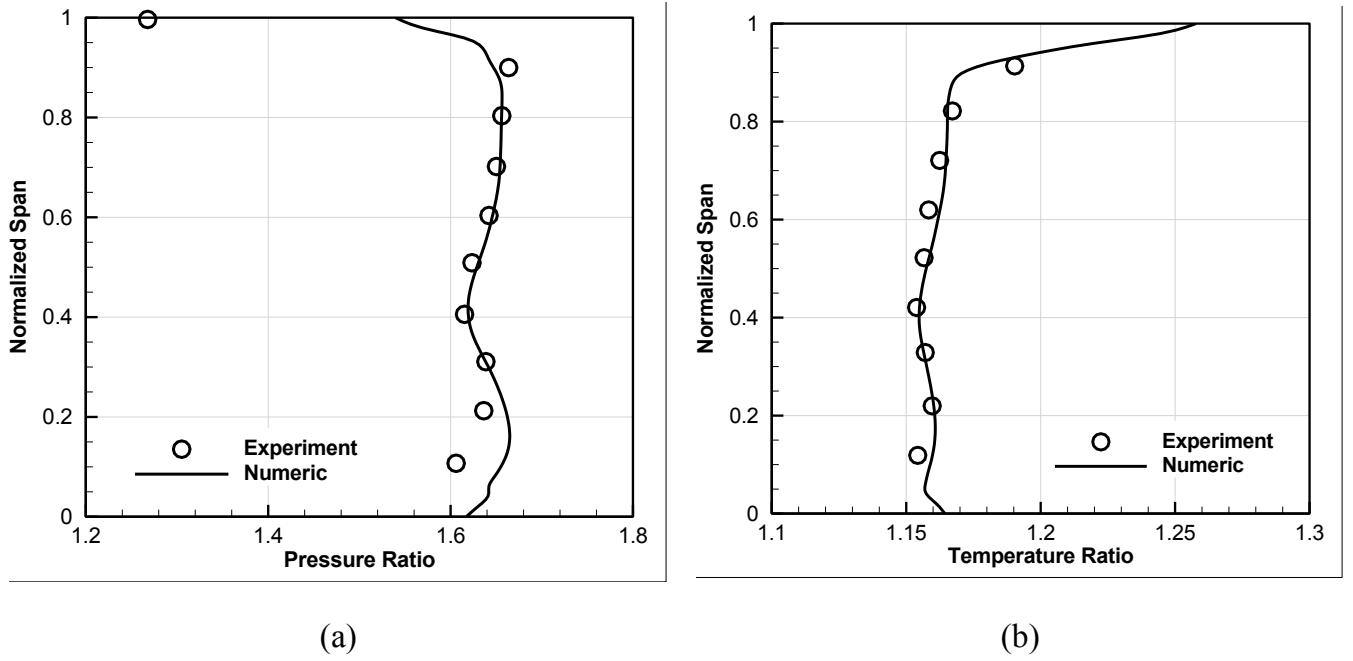


Fig. 7. Radial profile validation of performance parameters at near peak efficiency flow for (a) pressure ratio, (b) temperature ratio

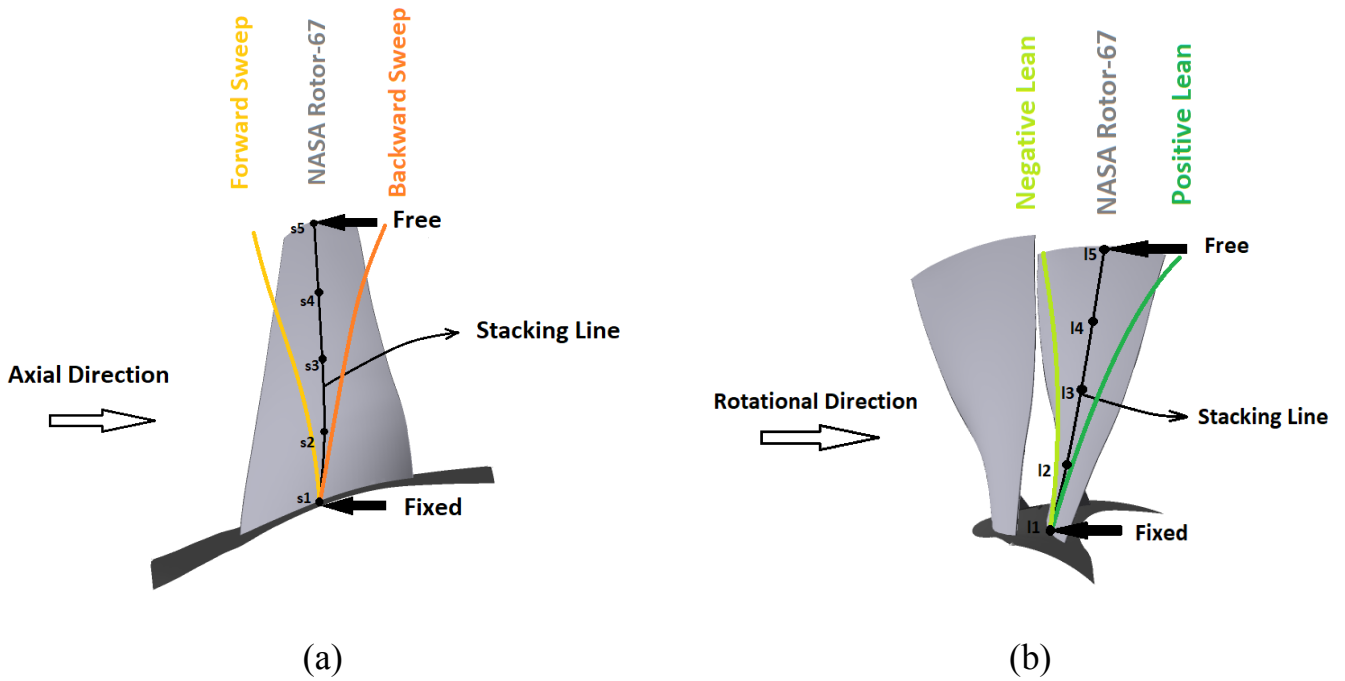


Fig. 8. Rotor parameterization for (a) sweep and (b) lean blades with 5 control points

Table 3. Coefficient bounds for leaned and swept blade

	Lean					Sweep				
Bound	l_1	l_2	l_3	l_4	l_5	s_1	s_2	s_3	s_4	s_5
Lower	-0.05	-2	-2	-2	-2	-0.05	-2	-2	-2	-2
Upper	+0.05	+2	+2	+2	+2	+0.05	+2	+2	+2	+2

first control point of the spline algorithm has a small range of changes. This small range between the upper and lower bounds of the first variable makes the stack line perpendicular to the hub surface.

3- 2- Constraints

If the curvature of the stacking line changes several times, the blade will encounter with high-stress concentrations. The number of curvature changes is limited to one as a constraint to avoid this issue. Therefore, for each of the lean and sweep splines, the second derivative is calculated, and the number of its sign changes is counted (Eqs. (1) and (2)).

$$n_{lean} = \frac{d^2 lean}{ds^2} \leq 1 \quad (1)$$

$$n_{sweep} = \frac{d^2 sweep}{ds^2} \leq 1 \quad (2)$$

Table 3 indicates the upper and lower bounds of spline algorithm control points for lean and sweep. The new stacking line and three-dimensional geometry are created using the spline algorithm's coefficients.

3- 3- Objective functions

The objective functions for the optimization of the rotor blade are to select the performance parameters of the compressor (mass flow rate ($\dot{m}$)) isentropic efficiency (η), and the total pressure ratio (PR)) at its design point. It uses the Eqs. (3) and (4) for the isentropic efficiency and the pressure ratio.

$$\eta = \frac{\left(\frac{P_{0out}}{P_{0in}}\right)^{\frac{(k-1)}{k}} - 1}{\left(\frac{T_{0out}}{T_{0in}}\right) - 1} \quad (3)$$

$$PR = \frac{P_{0out}}{P_{0in}} \quad (4)$$

3- 4- Optimization methodology

Neural network coupling with optimization tools is a common method to improve turbo-machine performance that has been mentioned in the research of Heidarian et al for the squealer-tip of an axial compressor [1], Tasharrofi et al [14] for a double-splitter centrifugal compressor, Ekradi et al. [15] for a centrifugal compressor, and Kamari et al. for airfoil [16]. The most important advantage of this optimization coupling is the reduction of the computational costs of generation in the optimization algorithm. So, the optimization procedure (including a genetic algorithm coupled to the artificial neural network) is a powerful tool to search for the optimal coefficients of desired data.

3- 4- 1- Artificial neural network

The generated database, consisting of 80 feasible cases, is used. The CFD has been solved to extract the compressor performance data. Using this database, three artificial neural networks have been trained. These artificial neural networks (ANNs) are trained using the database to predict the objective functions.

The ANNs consist of hidden layers; each layer involves neurons. Also, the process of the training convergence of artificial neural networks is illustrated in Fig. 9. The details of ANN settings are given in Table 4. The verification of ANNs with the CFD results of the database is done and presented in Fig. 10.

In Fig. 10, the generated databases and predicted results from neural networks are simultaneously plotted for benchmarking accuracy. As inferred, minor differences in the initial data results are observed, which practically converge to zero with increased database data, providing a very suitable validation tool as a practical instrument in the mathematical optimization of compressors.

3- 4- 2- Genetic algorithm

Now, the Genetic algorithm is used to find the optimum of the objective function. In the optimization loop, the ANNs predict the objective functions. For the optimum point, the flow domain is solved by CFD and validated with the ANNs. With the specified tolerance mentioned in Table 5, until the convergence criterion of the optimization loop has satisfied. Throughout this process, the CFD results are added to the

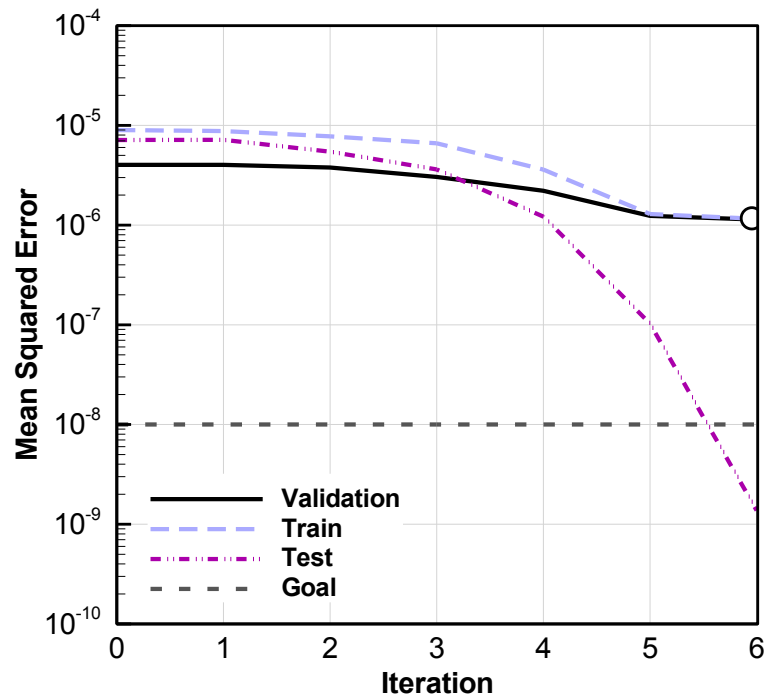


Fig. 9. Training convergence of ANN

Table 4. Details of the Artificial Neural Networks

Criteria	Function/ Value
Number of hidden layers	3
Number of neurons	6
Feeding method	Backpropagation
Number of epochs	200
Convergence goal	1e-8
Output function	Liner function
Neuron activation function	sigmoid
Data division	Random
Training algorithm	Levenberg-Marquardt
Performance	Mean squared error

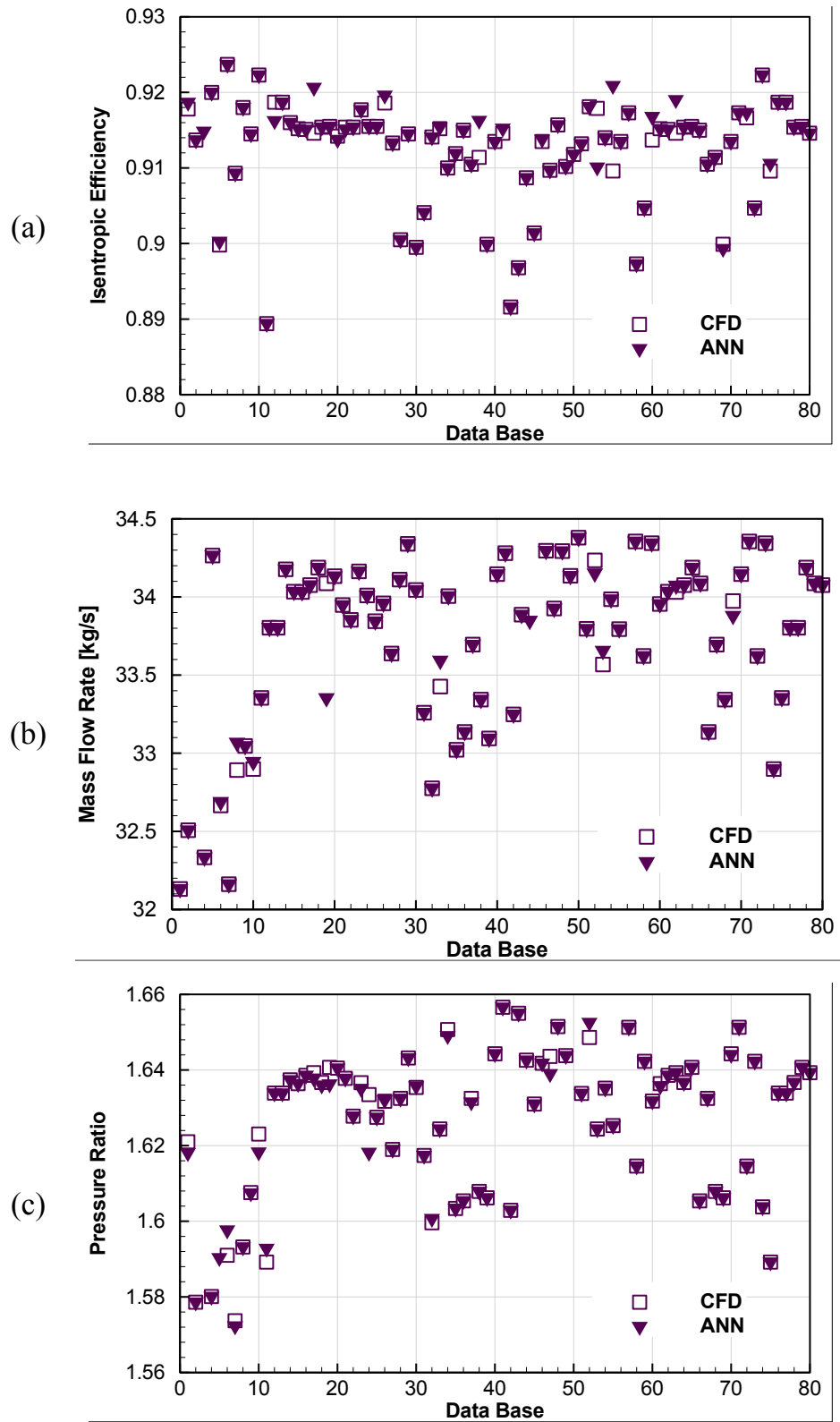


Fig. 10. Verification of Artificial Neural Network with CFD results for (a) isentropic efficiency, (b) mass flow rate, and (c) pressure ratio

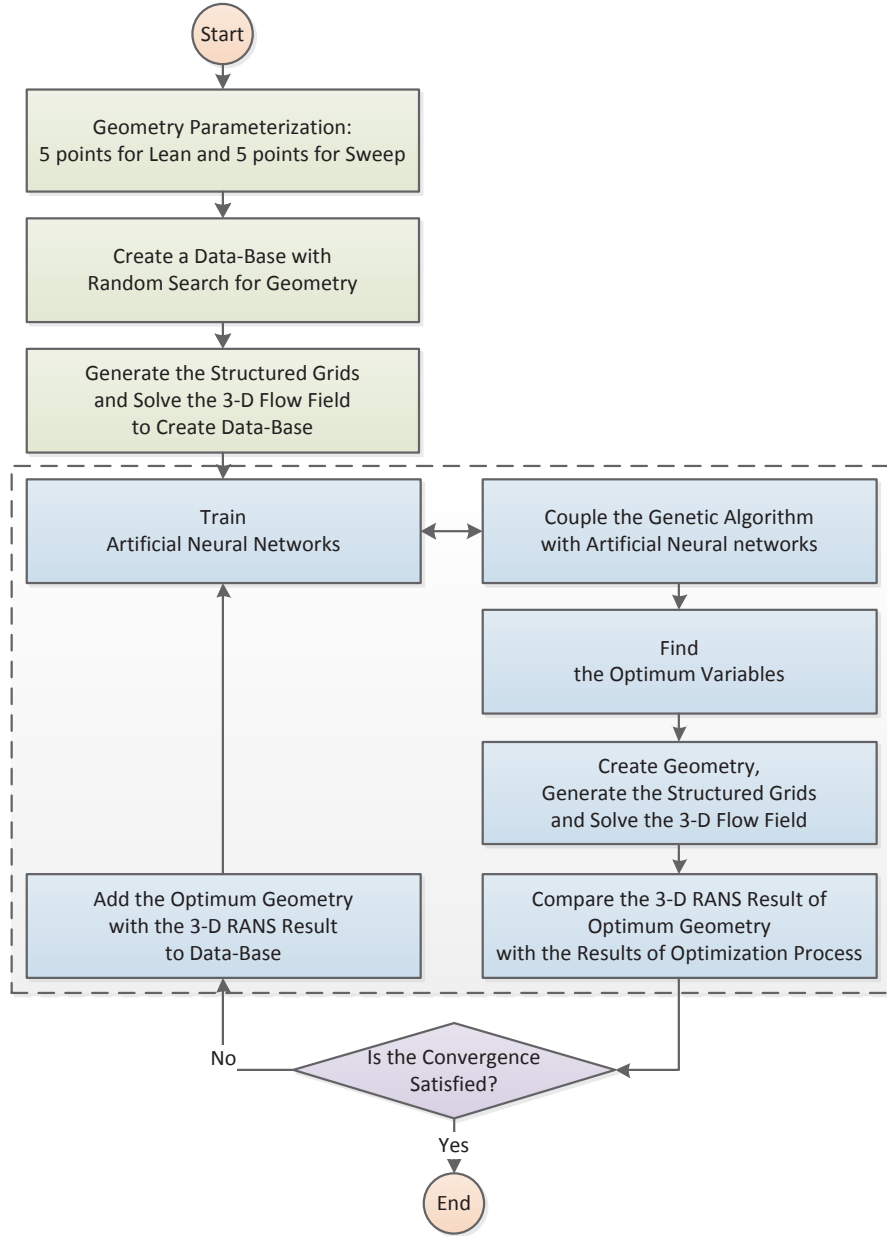


Fig. 11. Optimization procedure flowchart

database and the ANNs can predict better than the last result. The optimization flowchart is presented in Fig. 11, and settings are given in Table 5.

3- 4- 3- Fitness function and penalty

The genetic algorithm finds the minimum of the objective function in the first problem to increase the compressor efficiency, mass flow rate, and pressure ratio; the objective function is defined as Eq.(5).

$$Obj = (-PR) + (-\eta) + \left(-\frac{\dot{m}}{\dot{m}_D}\right) \quad (5)$$

Some parameters may increase while the targeted efficiency may be decreased. Because the objective functions may conflict with each other. To isentropic efficiency is defined as the objective function to handle this issue. The penalty function, Eq.(6), and the objective function, Eq.(7), are defined.

$$Penalty = \max(0, (\eta_D - \eta)) \quad (6)$$

$$Obj = (-PR) + \left(-\frac{\dot{m}}{\dot{m}_D}\right) + (10^3 \times Penalty) \quad (7)$$

Table 5. Details of the Genetic Algorithm

Criteria	Function / Value
Population Size Data	20
Generations Number	100
Convergence Criteria	1e-6
Mutation	Uniform (0.1)
Selection	Uniform
Crossover	Two-point Crossover
Fitness Scaling	Rank

3- 5- Feasible bounds

The results of the increase (or decrease) in objective functions of shape-changing compressors with simultaneous lean and sweep modifications have been evaluated and depicted in Fig. 12.

According to the figure, the initial databases and optimized geometries extracted in the genetic algorithm process are illustrated, indicating the flexible areas for performance changes in compressors. The two superior geometries selected based on the optimization process constraints, named PR-Improved and Best-Optimized, have maintained the previous isentropic efficiency while maximizing the increase in pressure ratio and mass flow rate. These two geometries are detailed and explained in the results.

4- Result and discussion

The results of three-dimensional analysis of axial compressor flow and optimized rotors have been processed, leading to improvements in the design point results. It is worth mentioning that the improvements are not only evident at the design point but also show relative enhancements in other off-design conditions, highlighting the utilization of these characteristic curves in the thermodynamic cycle modeling.

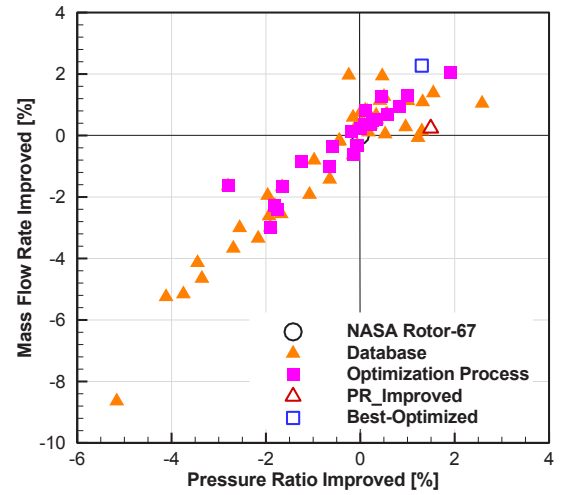
4- 1- Characteristic maps

After reaching the optimized rotors that demonstrated compatibility with the imposed constraints and showed relative improvements at their design point, the characteristic curves of their design speed have been completed and compared with each other in Fig. 13.

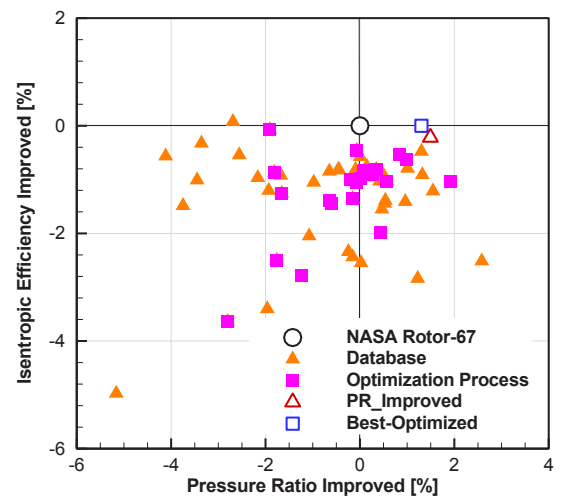
The results indicate that both geometries of the optimized rotors have shown improvements in mass flow rate at the design point and other points close to the stall regions. In the best rotor, the improvement in performance efficiency compared to the base geometry has been striking, and these values are presented in Table 6.

The pressure ratios of the PR-Improved and Best-Optimized rotors have improved by 1.486 % and 1.297 %, respectively.

(a)



(b)



(c)

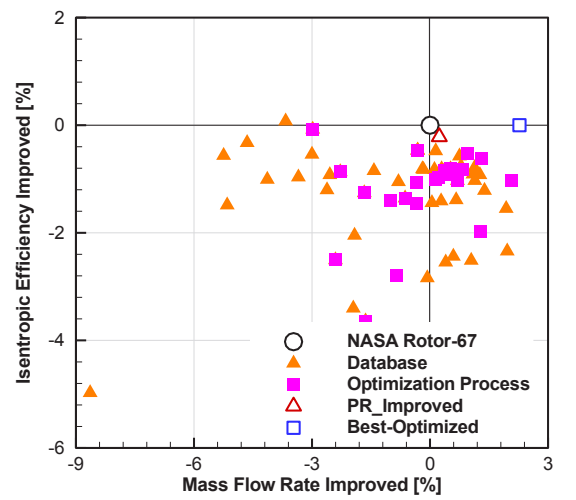


Fig. 12. Comparison of the improvement of optimized rotor and NASA Rotor-67 for (a) pressure ratio-mass flow rate, (b) pressure ratio-isentropic efficiency and (c) mass flow rate-isentropic efficiency

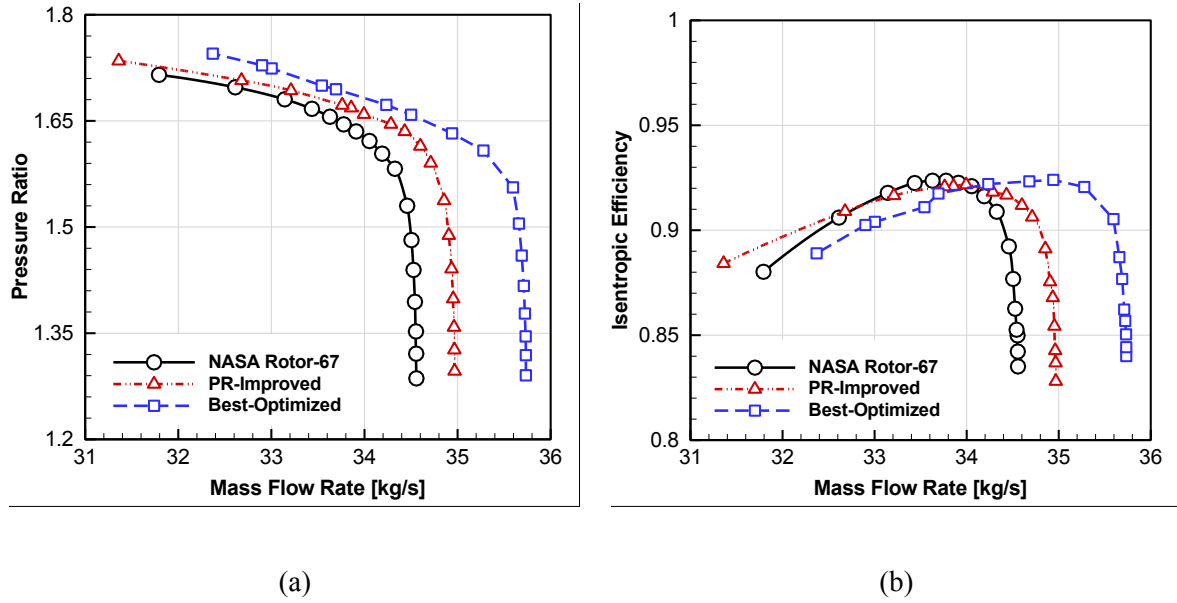


Fig. 13. Comparison of the optimized rotor and NASA Rotor-67 for (a) pressure ratio-mass flow rate and (b) isentropic efficiency-mass flow rate

Table 6. Performance parameters and its improvement for the optimized rotor at design, choke, and stall point

point	Case	Performance parameters			% Improved objective functions		
		Pressure ratio	Isentropic efficiency	Mass flow rate [kg/s]	Pressure ratio	Isentropic Efficiency	Mass flow rate
Choke	Rotor-67	1.286	0.835	34.558	---	---	---
	PR-Improved	1.296	0.828	34.968	0.801	-0.843	1.188
	Best-Optimized	1.291	0.840	35.733	0.334	0.589	3.403
Design	Rotor-67	1.635	0.923	33.91	---	---	---
	PR-Improved	1.6594	0.921	33.99	1.486	-0.098	0.245
	Best-Optimized	1.6563	0.923	34.681	1.297	0.174	2.020
Near-stall	Rotor-67	1.715	0.880	31.793	---	---	---
	PR-Improved	1.735	0.884	31.360	1.149	0.454	-1.362
	Best-Optimized	1.745	0.889	32.370	1.746	1.000	1.815

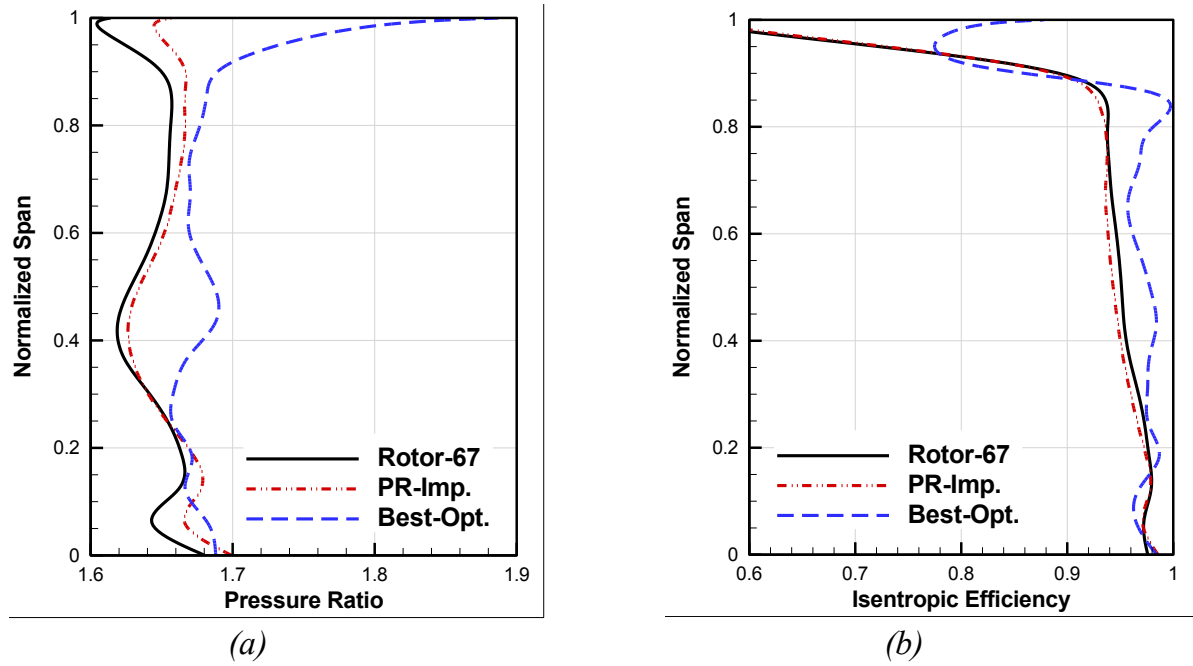


Fig. 14. Comparison of the radial distribution of optimized rotor and NASA Rotor-67 for (a) pressure ratio and (b) isentropic efficiency

Table 7. Surge Margin and its improvement for the optimized rotors Performance parameters and its improvement for the optimized rotor at design, choke, and stall point

Case	Surge Margin	
	Surge Margin	Improved [%]
Rotor-67	0.106	---
PR-Improved	0.118	10.727
Best-Optimized	0.112	5.128

respectively. The isentropic efficiency of the PR-Improved decreased by only 0.1%. By adding a penalty and re-optimization, the optimal rotor with a relative improvement of 0.17% efficiency is obtained.

The Best-Optimized rotor has the most significant increase in mass flow at the design point, 2.020 %. The choking mass flow of the optimal geometries has also improved as well as its design point, which is another inferred result of the characteristic curve. This improvement in choking mass flow of the Best-Optimized rotor is 3.403% compared to the Rotor-67.

4- 2- Surge margin

As the objective functions are improved at the design point, the surge margin can also be calculated to evaluate the improvement in the optimal geometry relative to the reference geometry. Eq.(8) is presented to calculate the surge margin [17].

$$SurgeMargin = 1 - \left(\frac{PR_D}{PR_{NS}} \right) \times \left(\frac{\dot{m}_{NS}}{\dot{m}_D} \right) \quad (8)$$

The surge margin and its enhancement percentage are calculated and presented in Table 7.

The research study [1] [14] shows that a positive sweep has more surge margin than a negative sweep. In this study, the surge margin enhancement for the first and second optimal compressors equals 10.7% and 5.1 %.

4- 3- Radial profile

Comparison of hub-to-tip distribution of pressure ratio and isentropic efficiency parameters in optimized rotors relative to the Rotor-67, extracted with the help of circumferentially averaged mass distribution, is presented in Fig. 14.

According to these results, improvements in isentropic efficiency performance are clearly observed in near-design tip speeds (above 80% span), especially in the geometry of the best rotor, which has been swept downstream in the downstream direction of the flow.

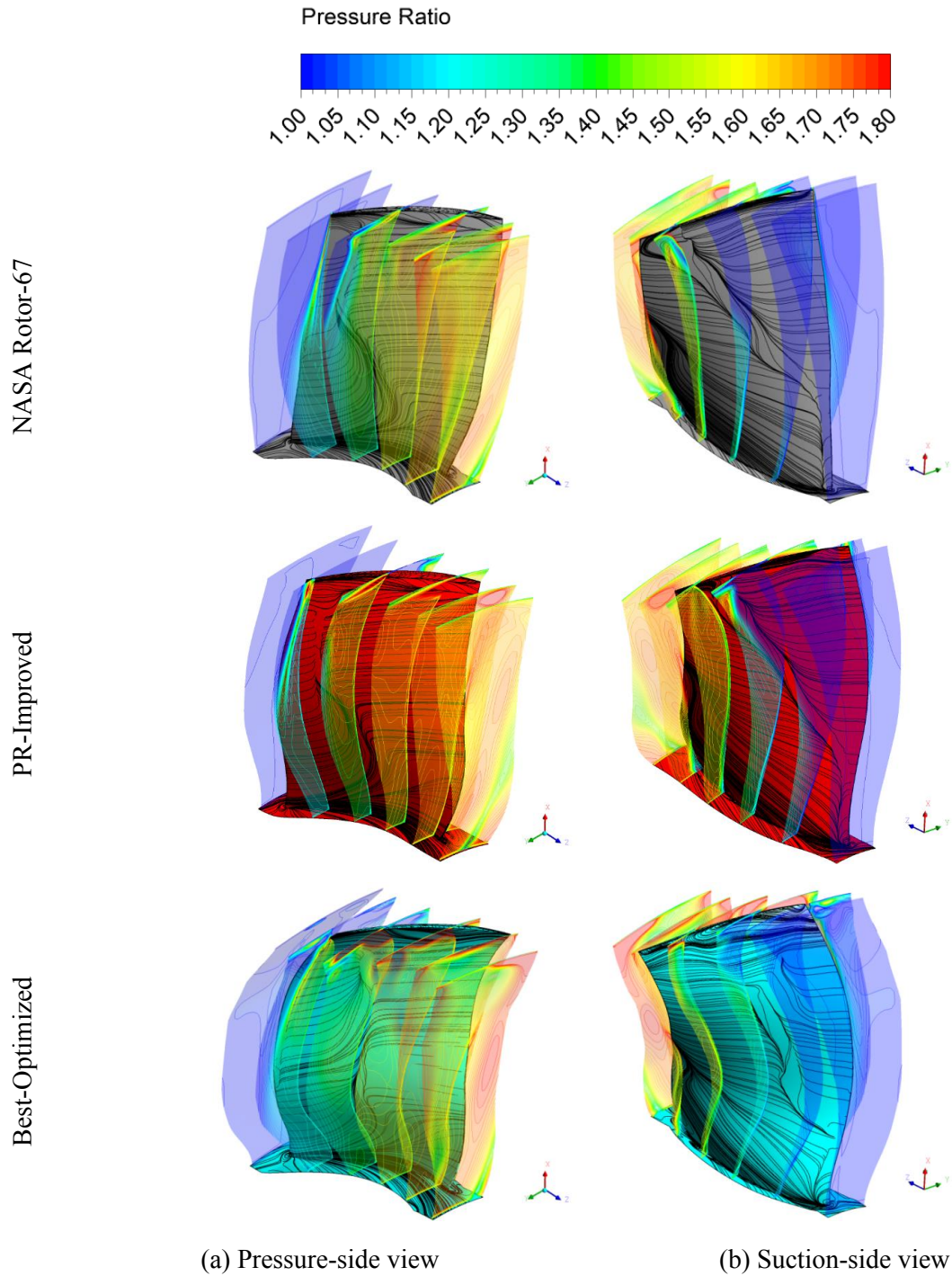


Fig. 15. Comparison of pressure contour at (a) pressure-side and (b) suction-side for: NASA Rotor-67, PR-Improved rotor and Best-Optimized rotor

The noticeable difference seen for the pressure ratio and isentropic efficiency ratio for Best-Optimized compared to the tip region geometry is due to the backward sweep that was obtained in extracting a superior rotor based on AI model, resulting in higher performance.

4- 4- Pressure loading effects

4- 4- 1- Streamline distribution and shock wave

The distribution of pressure ratio of optimized rotors compared to the baseline geometry, in two pressure and suction sides, for comparison, is illustrated in Fig. 15 (Rotor-67 with grey, PR-Improved with red, and Best-Optimized with blue

colors)

According to the contour results, the location of the normal shock wave occurrence in the baseline rotor has been converted to a more inclined shock in the optimized rotors. Therefore, as expected, lower pressure drop and higher pressure loading can be achieved in the optimized rotors, leading to optimal compressor performance.

As is apparent, the spatial shape of the shock is influenced by the blade curvature [8]. So, the pattern of the blade streamline at the normal shock locations (as mentioned) causes the flow separation. However, this occurrence is directed downstream in the optimal rotors, and therefore, it has reduced the flow separation and improved the compressor's performance.

4- 4- 2- Blade pressure loading

The pressure surface distribution of the optimal rotor surfaces is also extracted and calculated compared to the baseline rotor and presented in Fig. 16 from the leading edge to the trailing edge.

Thus, improving compressor performance can be achieved by displacing the normal shock to an inclined shock, which is due to the direct effects of the backward sweep three-dimensional compressor geometry.

Furthermore, the pressure loading of the Best-Optimized rotor, although seen to be higher than the two other rotors in its pressure surface, has experienced delayed flow separation on its suction surface, leading to better performance on the performance curve.

4- 5- Mach number distribution

Fig. 17 displays the distribution of relative Mach number along the streamwise of the optimized rotors compared to Rotor-67 in a contour graphic form.

Attention to the distribution of Mach number along the entire flow path (from leading edge to trailing edge) suggests that loading resulting from simultaneous changes in lean and sweep has had more uniform effects on the distribution of relative Mach number along the flow path. Although relative Mach numbers close to the magnitude of 1.4 have been maintained near the tip region, it seems that in optimal rotors, the difference between the minimum and maximum values of these distributions is less visible. This issue has also been discussed and evaluated in the blade-to-blade section.

4- 6- Meridional-view Effects

Fig. 18 shows the distribution of meridional velocity for the two optimized rotors compared to Rotor-67. This velocity can be an indicator of uniformity and magnitude of speed in the aerodynamic field of the new rotors.

In the blade pressure loading section, it was observed that the two optimized rotors had higher pressure performance compared to Rotor-67, indicating a more uniform axial velocity distribution, covering a larger space of the streamwise aerodynamic field from hub to shroud. Therefore, the accuracy of this more uniform axial velocity distribution compared to Rotor-67 can be achieved as shown in Fig. 18.

4- 7- Blade-to-Blade Effects

The effects of lean and sweep can be investigated in altering the relative Mach contour and improving the incidence and deviation angles in three sections: hub, mid, and shroud. This is because changing the airfoil arrangement leads to changes in loading and displacement of these quantities, which can lead to improvements.

Fig. 19 shows the relative Mach contour comparison of the designed points of Rotor-67 and the optimal rotors for the three sections: hub, middle, and shroud.

The blade tip contour shows that the maximum Mach number (1.4) covers a larger area of the flow path between the two blades of the second optimal geometry; the shock profile is delayed, and therefore, a higher performance of the compressor is obtained at its design point. The mass flow of this compressor has also reached its maximum in comparison with the other two ones.

4- 8- Circumferential Effects

The applied changes in the optimal rotors lead to circumferentially aerodynamic effects, which in this section are discussed in terms of the velocity curl of optimized rotors compared to the baseline design.

4- 8- 1- Velocity Curl

The distribution of normalized circumferential velocity gradient in the tip region of the rotor blade is extracted just behind the rotor and presented in Fig. 20. These curls, which depict a measure of instability, are less in the optimal rotors at the optimum point compared to the baseline geometry.

The normalized velocity curl that was previously observed in the side wall of the tip region in Rotor-67 has shown a considerable decrease with changes in the circumferential direction, which can be an indicator of increased orderliness in the streamlines of the critical tip region. Its effects on the flow smoothness, increased performance range, and enhanced stability at the blade tip margin are clearly evident.

4- 8- 2- Entropy Generation

Fig. 21 illustrates the contour distribution of normalized entropy (per 100 [J/kg.s]) rotor outlet at the design point for two optimal geometries compared to Rotor-67.

It is deduced from the entropy contour distribution that for two optimal geometries, between 0% to 90% span, an almost uniform distribution with little change in entropy is observed (unlike Rotor-67). However, locally, the entropy of the optimal blade tips is higher than Rotor-67, which could be due to changes in the sweep tip and a relative increase in the Mach number. Nevertheless, the overall average output surface of the compressor in optimal rotors has greater uniformity.

4- 9- Geometry Comparison

The geometric changes that occurred in the optimization process for the optimized rotors compared to a baseline rotor have been illustrated in this section, including the spanwise and sweep angles of the sections depicted in the top view.

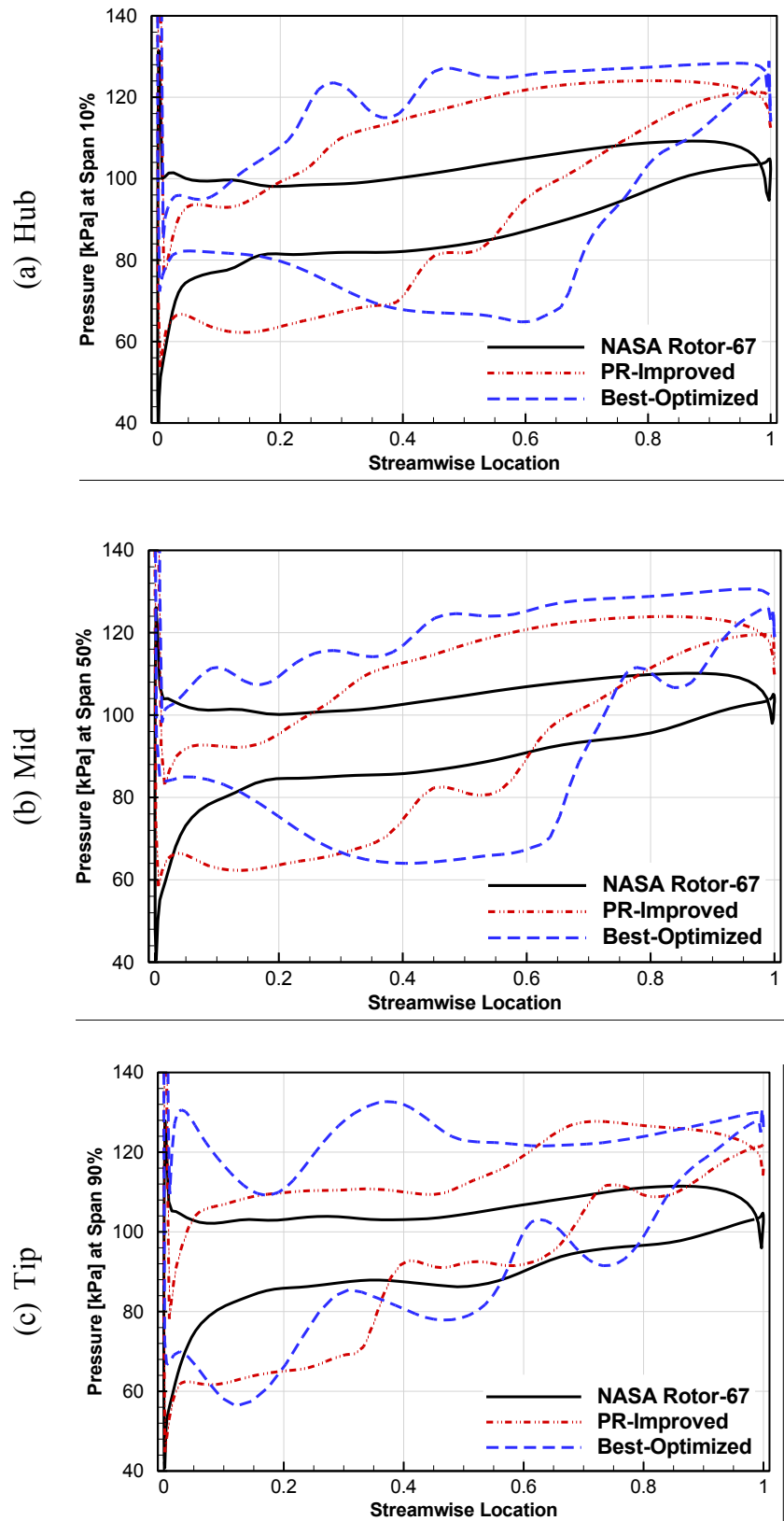


Fig. 16. Comparison of the pressure distribution of the optimized rotor at the (a) hub, (b) mid and (c) tip spans

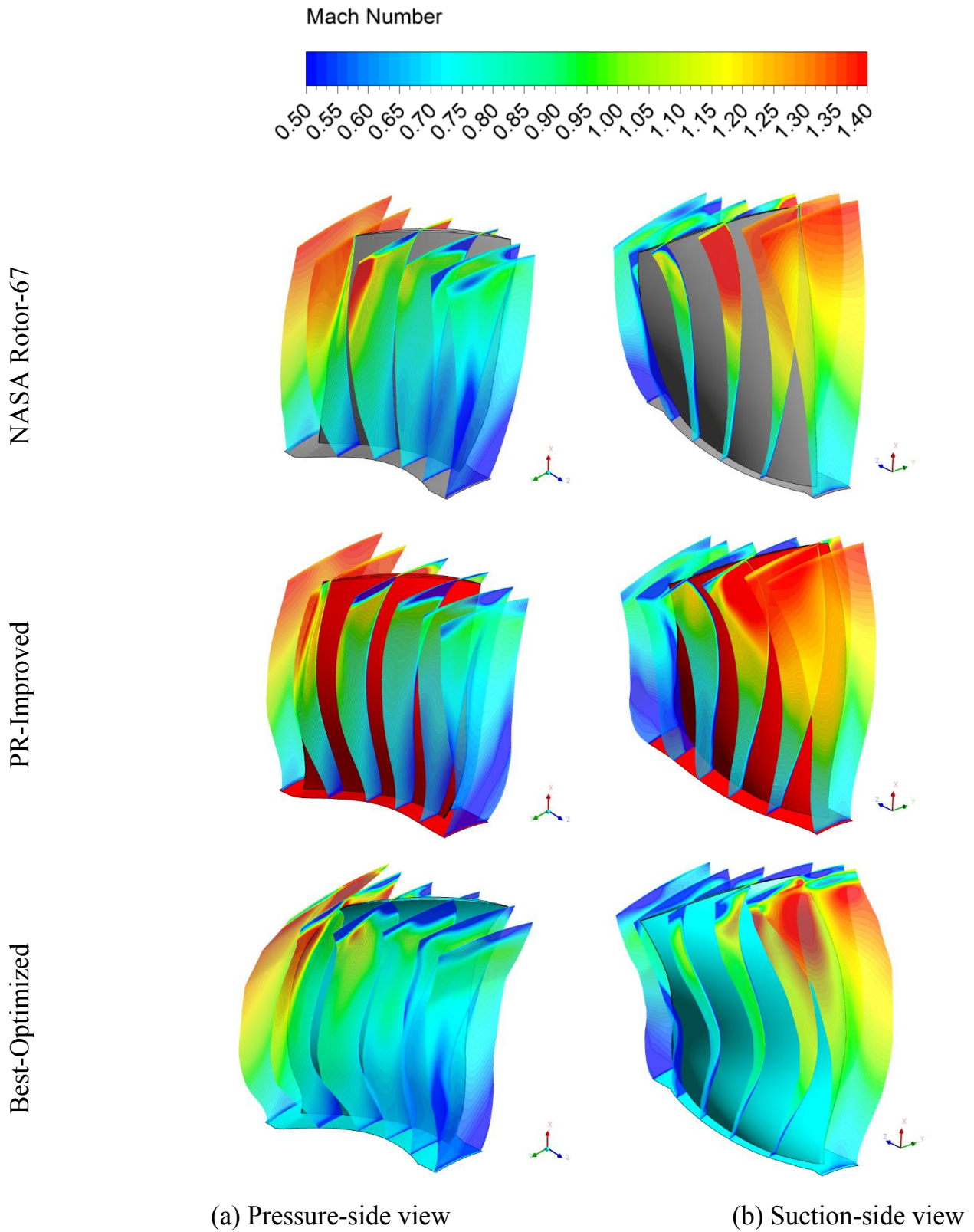


Fig. 17. Comparison of pressure contour at (a) pressure-side and (b) suction-side for: NASA Rotor-67, PR-Improved rotor and Best-Optimized rotor

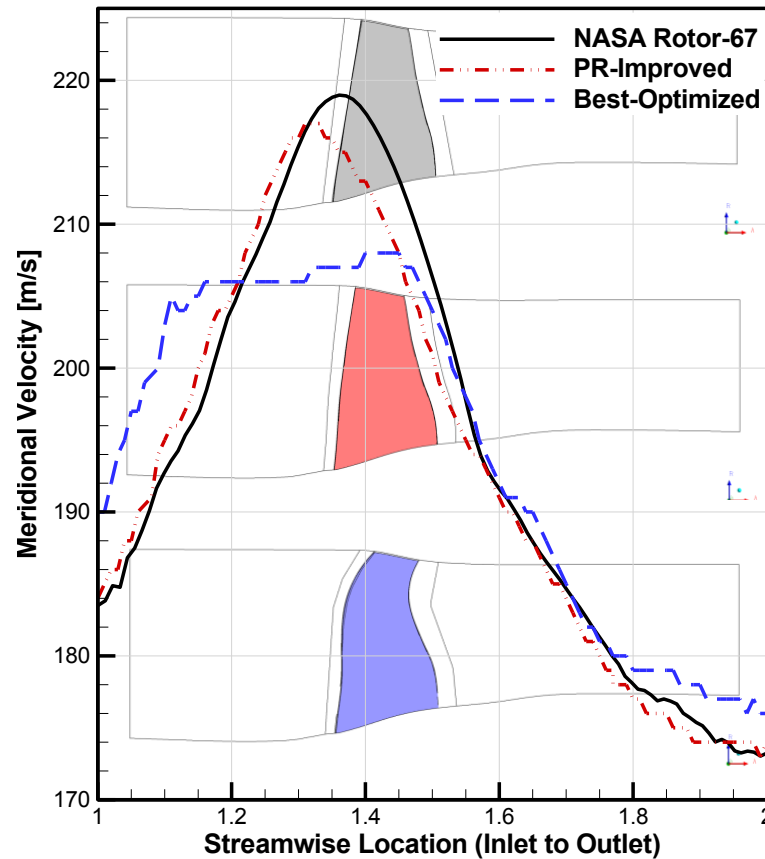


Fig. 18. Comparison of meridional velocity of optimized rotors with NASA Rotor-67

4- 9- 1- Stacking Line

Fig. 22 and Fig. 23 show the changes in the lean and sweep of optimal rotors compared to the baseline rotor for 3-D and 2-D views. The optimal lean and sweep coefficients are also presented in Table 8.

As seen in Table 8, the PR-Improved optimal rotor has set a forward sweep, while the Best-Optimized rotor initially has a forward sweep and then enters into tip back sweep. This behavior trend, which leads to maximum improvement in rotor performance, has been previously observed in the Benini research.

The lean behavior of both optimal rotors was the same, with a 40% change in incidence seen for optimal rotors (Best-Optimized rotor has more curvature in the span of about 30% than the PR-Improved rotor). The tip of the optimal rotors has also returned to the optimal airfoil range.

Now, the comparison of the optimization results of the other studies carried out regarding lean and sweep axial compressors is given in Table 9. In most of these studies, adding a penalty has improved the objective functions and prevented the reduction of other objective functions.

5- Engine Performance

The performance study models and analyzes the thermodynamic cycle of a turbojet engine in this part. The compressor is powered by the turbine, which extracts energy from the expanding gas passing through it. The engine converts internal energy in the fuel to kinetic energy in the exhaust, producing thrust [18]. The schematic view of the engine is shown in Fig. 24.

The in-house thermodynamic code has been developed for on-design and off-design performance calculations. The engine's characteristics are used from GasTurb software [20] which is close to the present test case. For the axial compressor, the Rotor-67 is assumed to be the first compressor stage, and the other multi-stage compressors are modeled with a certain pressure ratio and isentropic efficiency.

In this in-house code modeling, the gaseous fluid is assumed to be composed of water (H_2O), carbon dioxide (CO_2), oxygen (O_2), nitrogen (N_2), argon (Ar), and hydrogen (H_2). Therefore, the mass fraction, molar fraction, gas constant, and specific heat at constant volume and constant pressure are calculated using Eqs.(9), (10), (11), and (12).

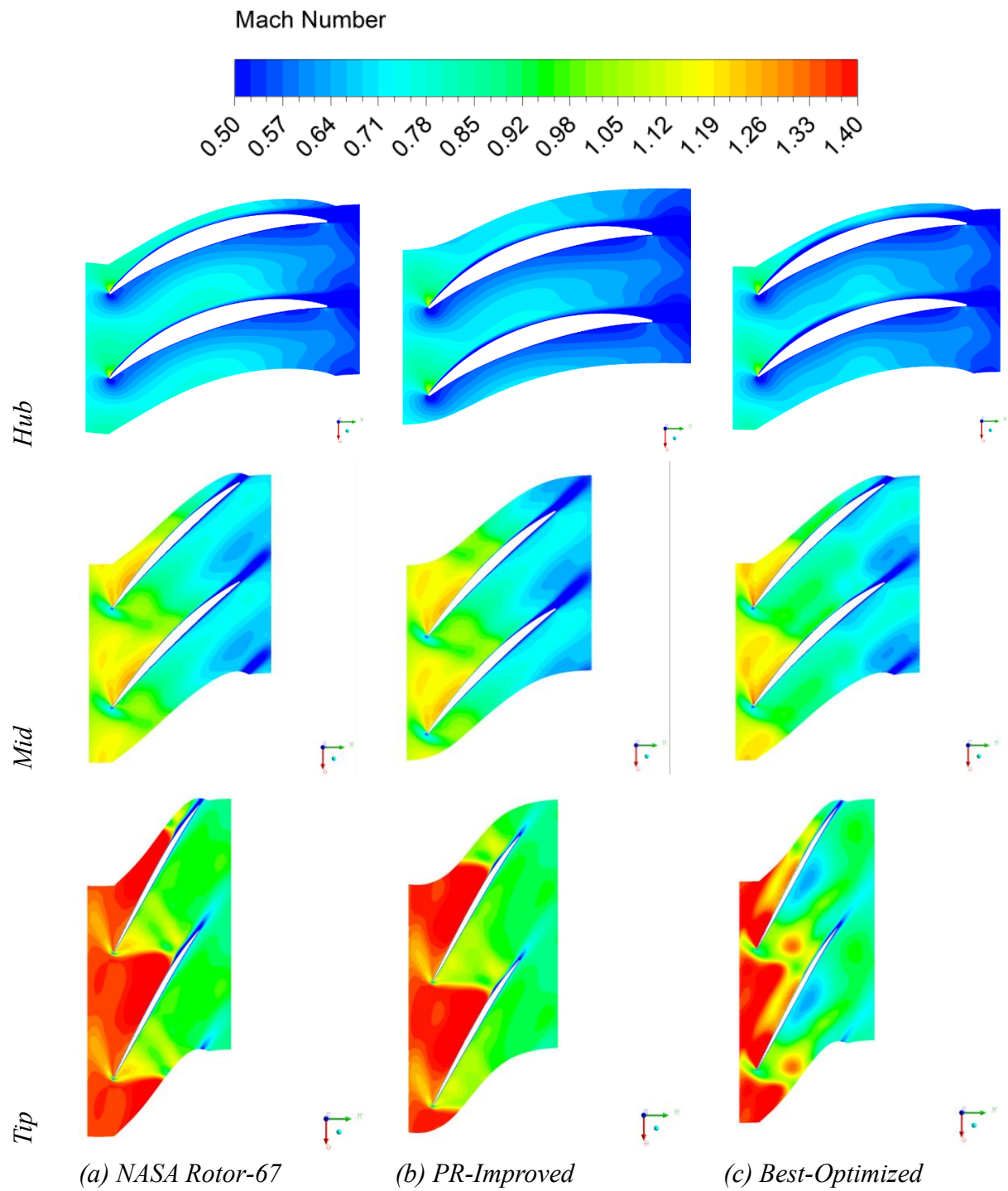


Fig. 19. Comparison of relative Mach number for: (a) NASA Rotor-67, (b) PR-Improved rotor, and (c) Best-Optimized rotor

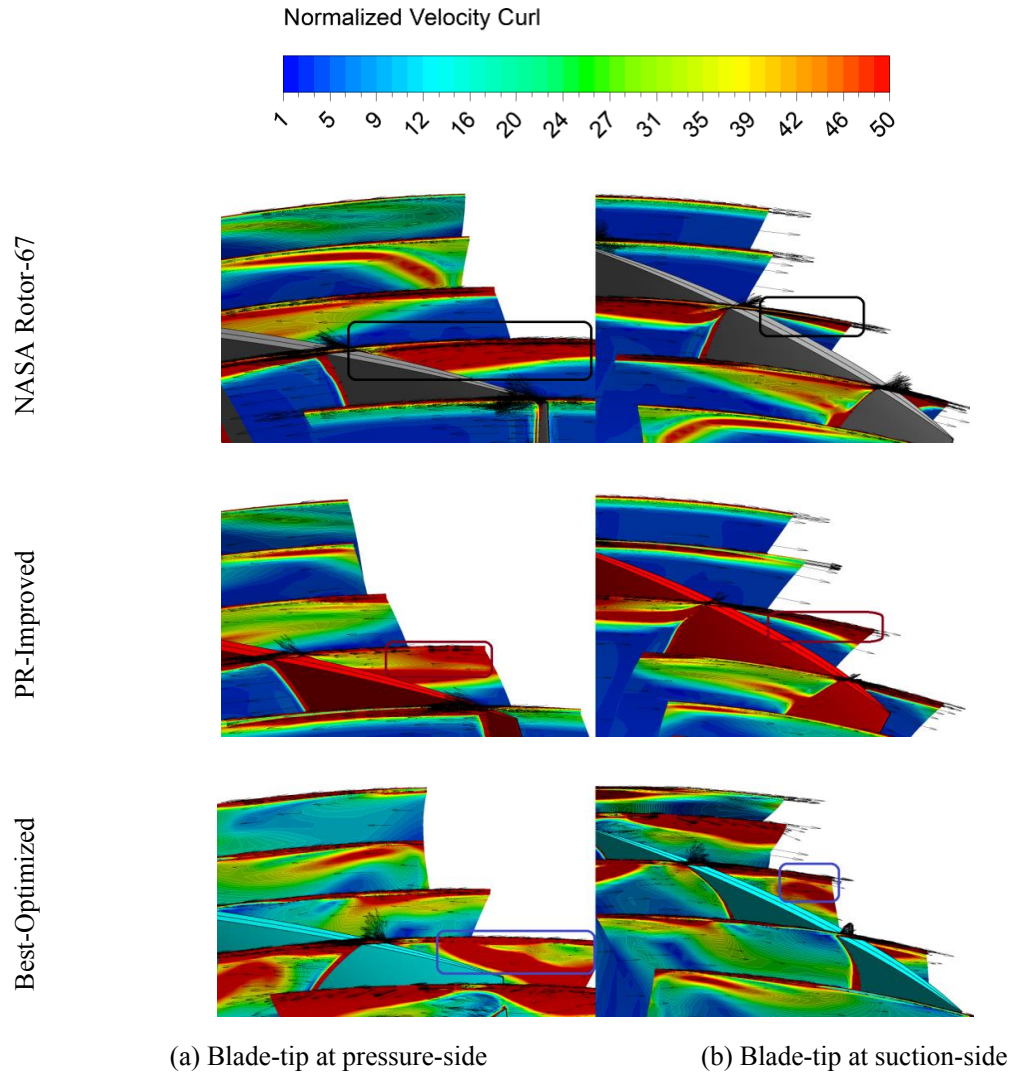


Fig. 20. Comparison of normalized velocity curl contour at the blade-tip of (a) pressure-side and (b) suction-side for: NASA Rotor-67, PR-Improved rotor and Best-Optimized rotor

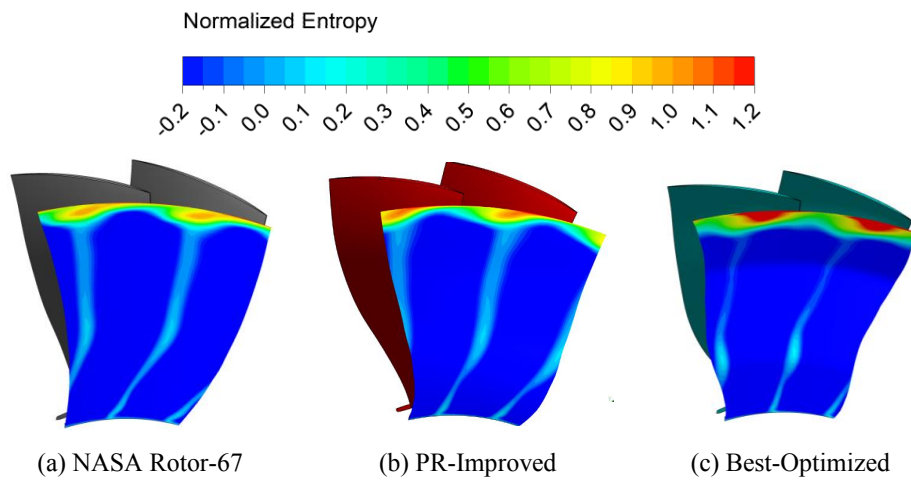


Fig. 21. Comparison of normalized velocity curl contour at the blade-tip of (a) pressure-side and (b) suction-side for: NASA Rotor-67, PR-Improved rotor, and Best-Optimized rotor

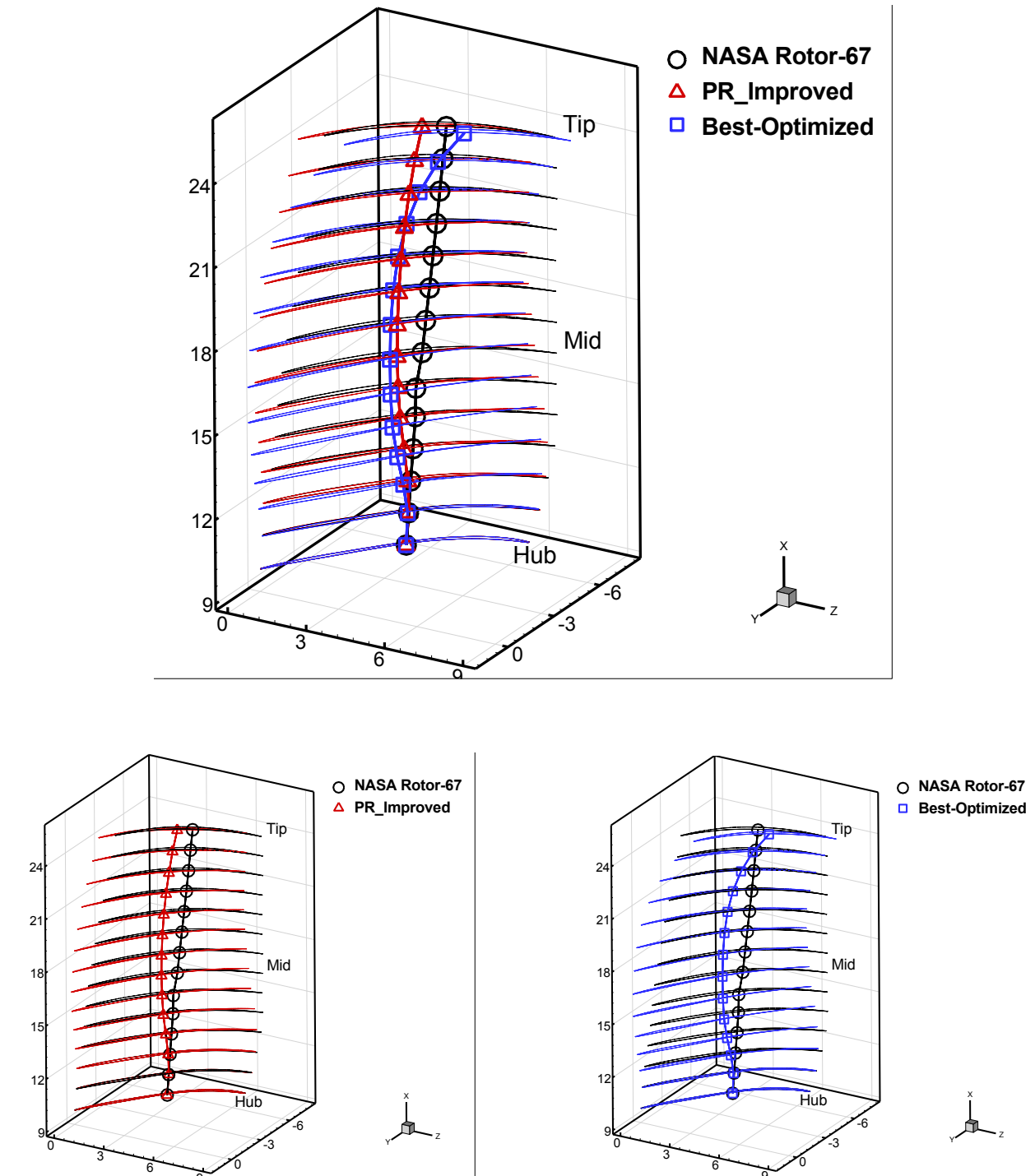
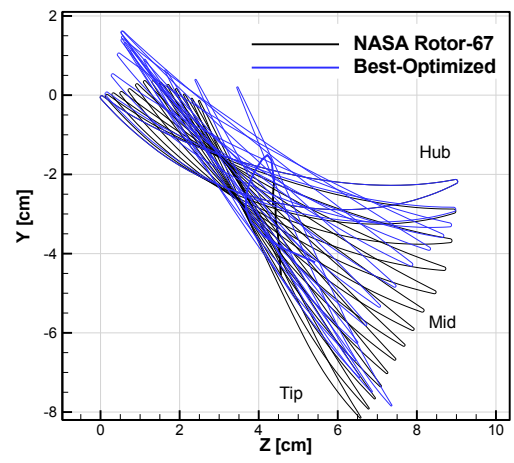
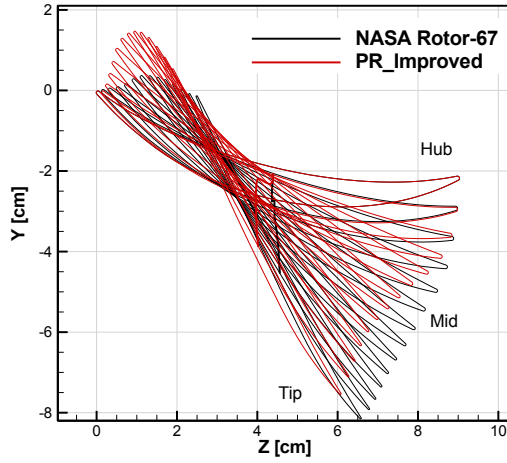
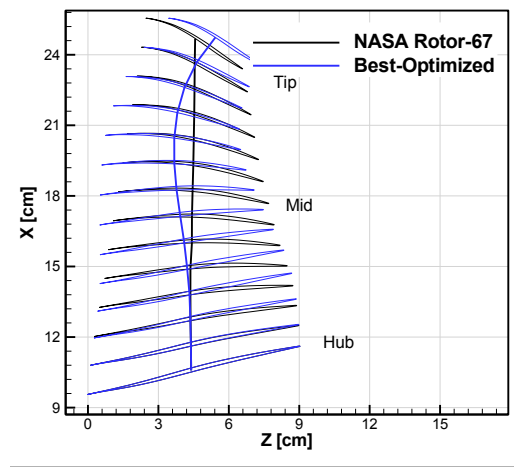
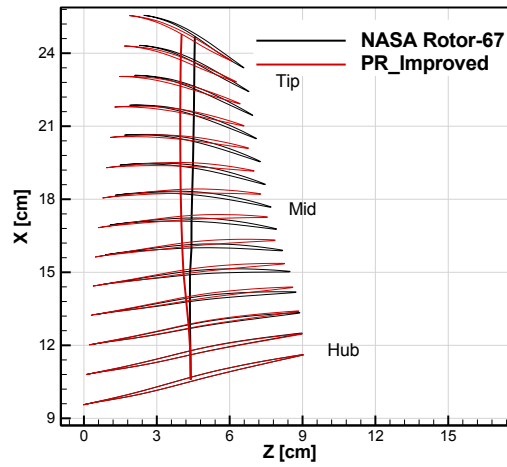


Fig. 22. Comparison of 3-D views of PR-Improved and Best-Optimized rotors with NASA Rotor-67

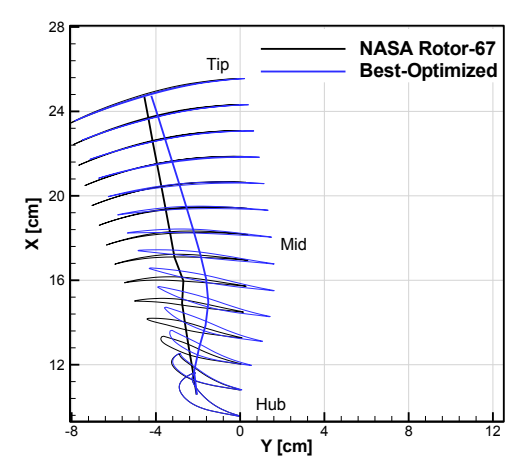
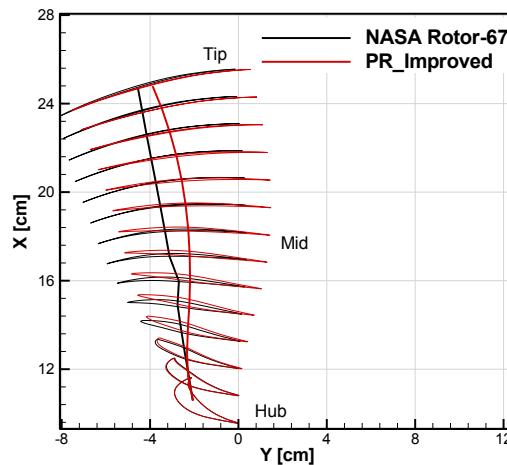
Top-View



Meridional-View



Front-View



(a)

(b)

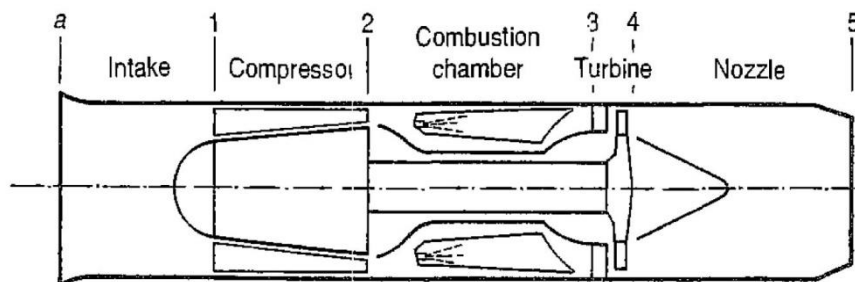
Fig. 23. Comparison of the top, meridional, and front views of (a) PR-Improved and (b) Best-Optimized rotors with NASA Rotor-67

Table 8. The lean and sweep coefficients of optimized rotors

Bound	Lean					Sweep				
	l_1	l_2	l_3	l_4	l_5	s_1	s_2	s_3	s_4	s_5
PR-Improved	0.048	-0.385	-1.003	-1.163	-0.671	-0.018	0.741	1.103	1.089	0.938
Best-Optimized	0.020	-1.087	-1.202	-0.775	-0.342	0.007	0.297	1.476	1.500	-1.498

Table 9. The brief of the lean and sweep study in comparison with present work

Study	Test Case	Pressure ratio increment	Mass Flow Rate increment	Isentropic Efficiency increment
Benini	Rotor-37	5.5 %	NA	-0.8 %
		0	NA	1.5 %
Oyama	Rotor-67	0.6 %	0.46 %	1.783 %
Samad	Rotor-37	1.62 %	NA	-0.04 %
		-1.55 %	NA	1.41 %
		1.25 %	NA	0.51 %
Wang X.D.	An industrial case	1 %	-0.04 %	1.1 %
Huang	Rotor-37	1.4 %	0.9 %	0.5 %
		0.1 %	0.2 %	1.2 %
		0	0.1 %	1.2 %
Cao	Rotor-37	0.05 %	NA	0.8 %
Present Work	Rotor-67	1.486	0.245	-0.098
		1.297	2.020	0.174

**Fig. 24. a single spool turbojet engine [19]**

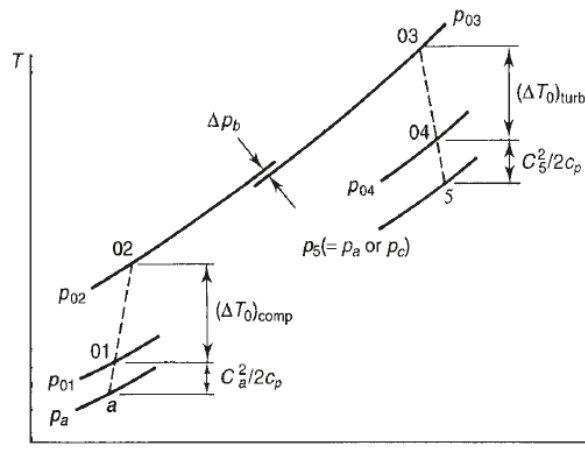


Fig. 25. The T-s diagram of a Turbojet cycle [18, 19]

$$MW_{mix} = \frac{1}{\sum_{i=1}^6 \frac{x_i}{MW_i}} \quad (9)$$

$$Y_i = x_i \times \frac{MW_{mix}}{MW_i} \quad (10)$$

$$R = \frac{R_{univ}}{MW_{mix}} \quad (11)$$

$$k = \frac{c_p}{c_v} = \frac{c_p}{c_p - R} \quad (12)$$

Once the mass fraction of each component is known, the enthalpy of the mixture is calculated by the Eqs. (13) and (14). The specific heat of constant pressure is a third-degree polynomial function of temperature, written in Eq.(15).

$$h_{mix} = \sum_{i=1}^6 h_i \times x_i \quad (13)$$

$$h_i = h_{ref} + \int_{T_{ref}}^T c_p dT \quad (14)$$

$$c_p(T) = a \times T^3 + b \times T^2 + T + d \quad (15)$$

These coefficients and the reference enthalpy of the species have been reported in [21].

5- 1- On-design Cycle

The thermodynamic on-design point analysis of this engine has been modeled ([19], [22]). This steady-state analysis of the engine was performed at ground-level conditions. Fig. 25 shows the engine thermodynamic cycle, and the on-design data are summarized in Table 10.

To evaluate the effect of compressor optimization in the design point conditions, the performance parameters of Rotor-67 and the best-optimized rotor (Best-Optimized rotor) are included in the turbojet cycle, and Thrust was subsequently calculated.

5- 2- Off-design Cycle

To investigate the effect of aerodynamic optimization of the first stage of the compressor on the off-design performance, the code for solving the equations of the turbojet engine has been developed. The process of off-design cycle analysis is discussed in detail in [19]. The compatibility equations are solved by the Newton-Raphson solver, and the Thrust [18] [19] [20] is computed. Fig. 26 shows the on-design and off-design processes.

The general characteristic diagrams are used for the compressor, turbine, and nozzle characteristic diagrams at the off-design process from 95% to 105% of rotational speed[23, 24]. Considering the Rotor-67 as the first compressor stage, Fig. 27 and Fig. 28 show the running line on the compressor's performance characteristics and turbine, respectively.

5- 3- Performance Result

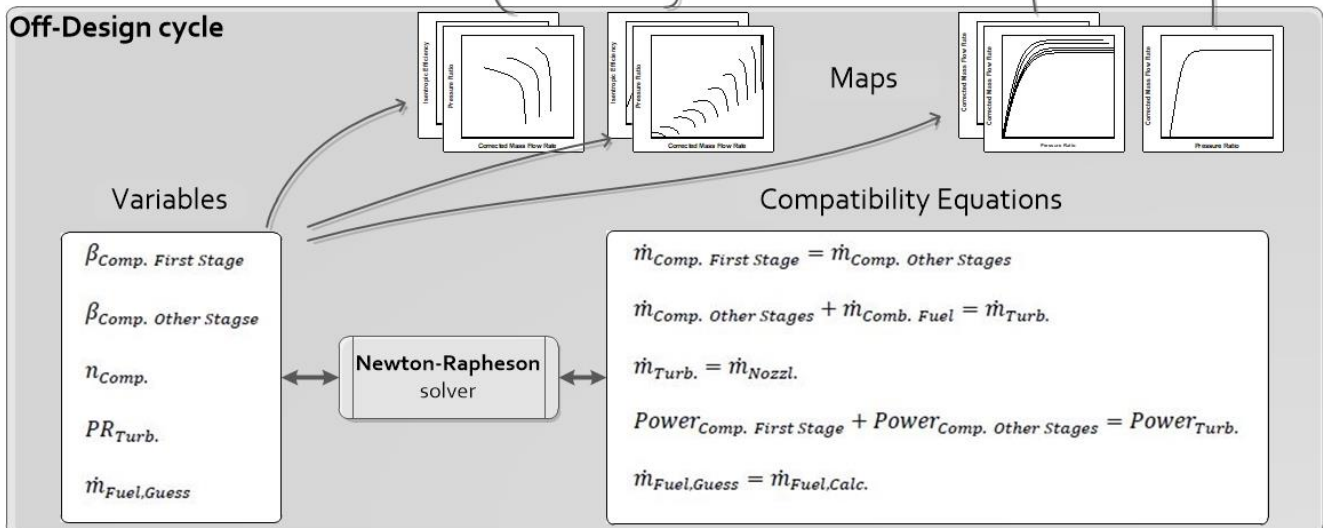
A turbojet engine is a machine that consists of the components of the first stage, compressor, turbine, and nozzle. The engine performance analysis is repeated for NASA-Rotor 67 and the Best-Optimized rotor as the first stage of the compressor. A comparison of running line results is given in Fig. 29. Also, the Thrust of the turbojet is shown in Fig. 30.

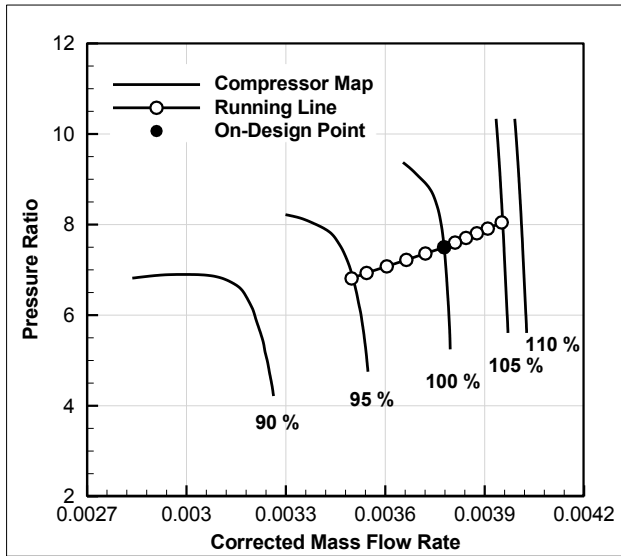
Table 10. The specifications of the turbojet engine

Component	Parameter	Unit	Value
Intake	Total Pressure Ratio	%	99
Compressor	First stage (Rotor-67) Total Pressure Ratio	---	1.635
	Isentropic Efficiency	%	92.26
	Other stages Total Pressure Ratio	---	7.34
	Isentropic Efficiency	%	85
Combustor	Burner Efficiency	%	99.5
	Burner Exit Temperature	[K]	1450
	Burner Pressure Ratio	%	97
	Fuel LHV	[MJ/kg]	43.124
Turbine	Isentropic Efficiency	%	89
Nozzle	Isentropic Efficiency	%	98

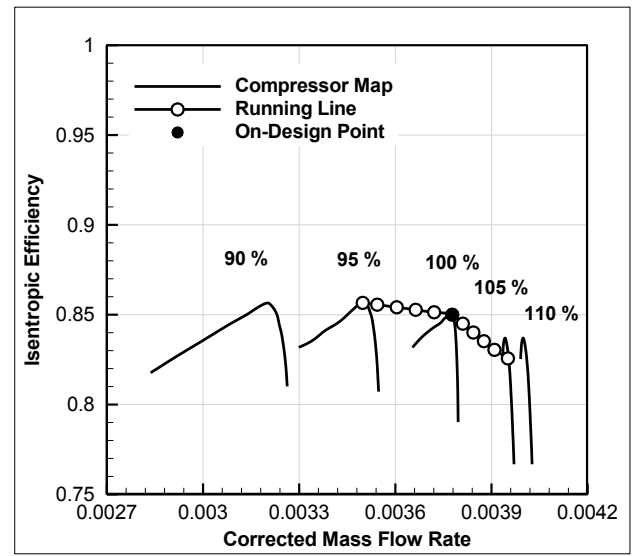
On-Design cycle

$m, P, h \rightarrow m, P, h \rightarrow m, P, h \rightarrow m, P, h \rightarrow m, P, h \rightarrow m, P, h \rightarrow m, P, h \rightarrow m, P, h$

**Off-Design cycle****Fig. 26. The on-design and off-design calculation process**

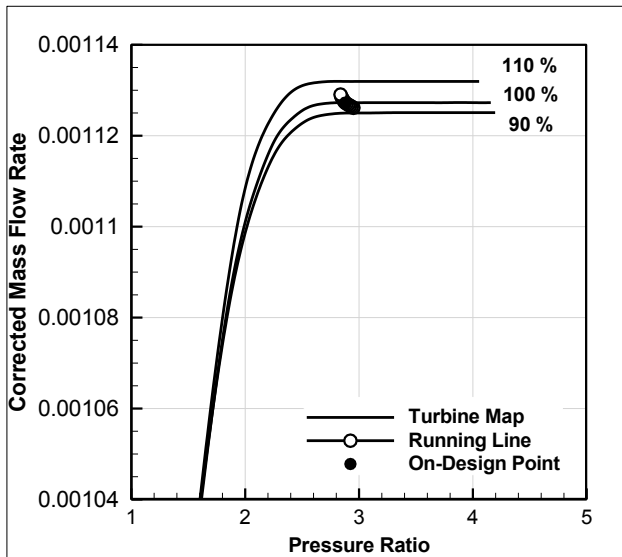


(a)

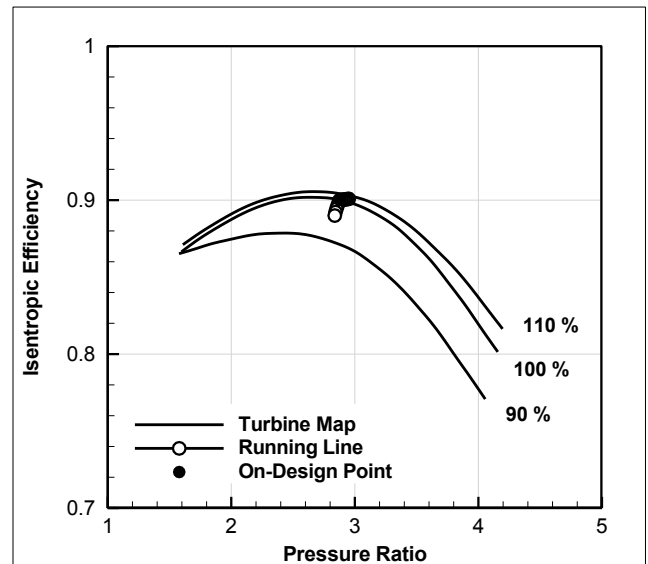


(b)

Fig. 27. The running line on the axial compressor map for (a) corrected mass flow rate-pressure ratio and (b) corrected mass flow rate-isentropic efficiency

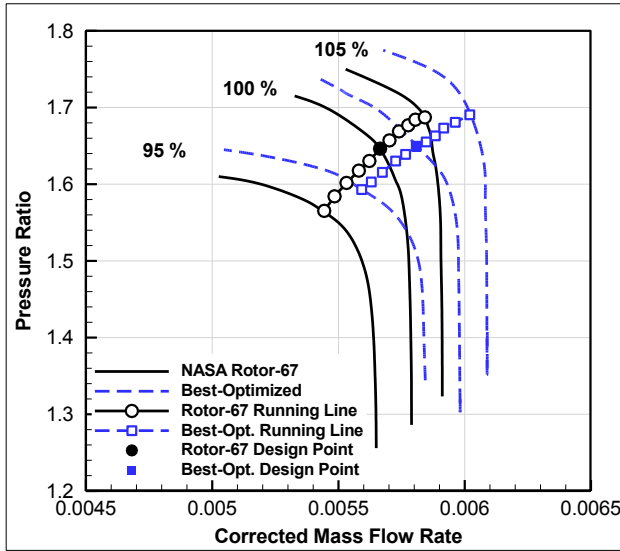


(a)

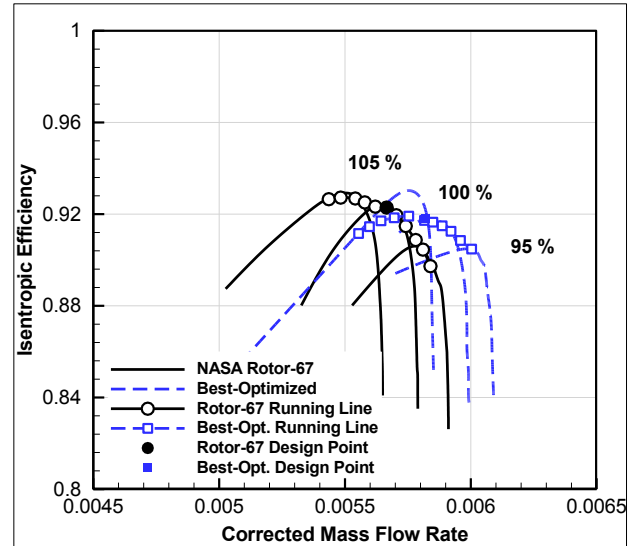


(b)

Fig. 28. The running line on the axial turbine map for (a) pressure ratio-corrected mass flow rate and (b) pressure ratio-isentropic efficiency



(a)



(b)

Fig. 29. Comparison of the running line of the Best-Optimized rotor with NASA Rotor-67 for (a) pressure ratio-corrected mass flow rate and (b) isentropic efficiency-corrected mass flow rate

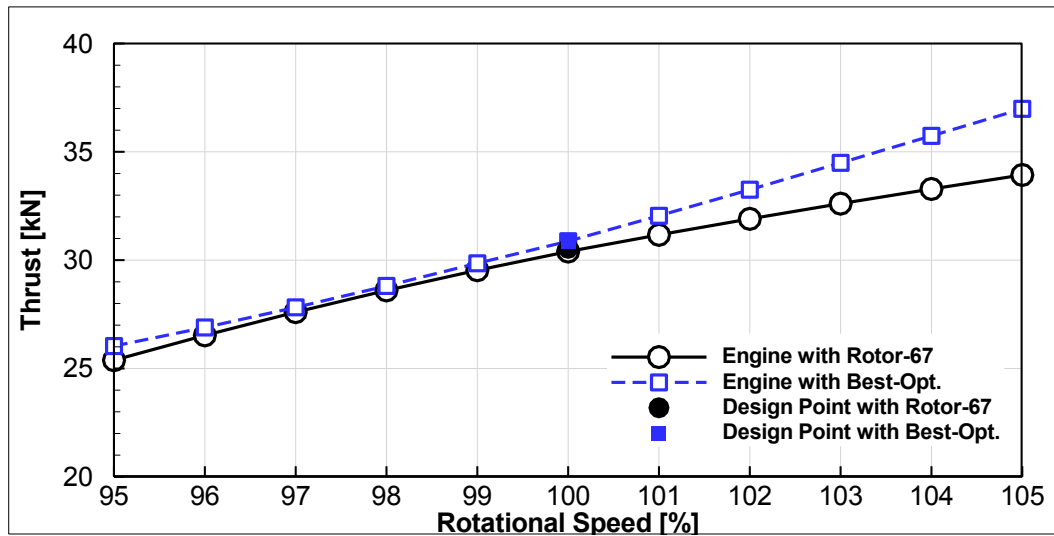


Fig. 30. Comparison of the Thrust of the Turbojet with NASA Rotor-67 and Best-Optimized rotor

The results show that the amount of Thrust has increased in all performance conditions. The percentage of thrust improvement at the design point is equal to 1.57 %.

6- Conclusion

The improvement of the axial compressor performance was performed at its characteristic design diagrams. The optimization of the problem was modeled by changing the

geometry of the stack-line by two spline curves for lean and sweep at the same time. The compressor has been simulated using CFD. This optimal process obtained two geometries optimized at the design point. The best-case results included improved mass flow and pressure ratios of 2.020% and 1.297%, while the isentropic efficiency did not decrease. The improvement of the surge margin is also one of the results obtained after optimization. The trend results of the streamline

behavior show that the negative sweep rotor directs the blade surface shock profile downstream, reduces the pressure loss, and increases the mass flow of the compressor.

Matching turbojet compatibility equations shows that the turbojet engine with Best-Optimized rotor as the first stage of the compressor improves the Thrust at the design point and off-design conditions. The improvement of Thrust is equal to 1.57 % at the design point.

Author Contributions:

Mojtaba Heidarian Shahri: Software, methodology, validation, formal analysis, data curation, investigation, visualization, writing—original draft preparation.

Ali Madadi: Project Administration, supervision, conceptualization, resources, data curation, writing—review, and editing.

Romina Ahadian: Software, methodology, validation, formal analysis, data curation, investigation, visualization, writing—original draft preparation.

All authors have read and agreed to the published version of the manuscript.

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Corresponding author:

Ali Madadi.

Nomenclature

a, b, d	Coefficients of c_p the equation
c_p	Specific heat capacity at constant pressure
c_v	Specific heat capacity at constant volume
Eq	Equation
h	Enthalpy
l_1, l_2, l_3, l_4, l_5	Control points of the spline algorithm for lean
$Imp.$	Improved

k	The ratio of specific heats
MW	Molecular weight
$\dot{m}$	Mass flow rate [kg/s]
NA	Not available
Obj	Objective function
$Opt.$	Optimized
PR	Pressure ratio
P	Pressure
R	Gas constant
s_1, s_2, s_3, s_4, s_5	Control points of the spline algorithm for sweep
T	Temperature
x	Mole fraction
y	Mass fraction

Subscripts

D	Design
in	Inlet
is	Isentropic
mix	Mixture
out	Outlet
NS	Near stall
0	Stagnation property
ref	Reference
$univ$	Universal

Greek symbols

η	Isentropic Efficiency
δ	Displacement

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