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Inverse Reconstruction of Absorption-Coefficient Distribution in a Two-Dimensional Radiative Medium from the Knowledge of Wall Heat Fluxes

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ABSTRACT: In this article, an inverse radiation analysis is presented to reconstruct the absorption coefficient distribution from the knowledge of wall heat fluxes for a two-dimensional, absorbing-emitting medium with black walls. The inverse approach aims to find the location of inclusion with a different absorption coefficient. For this purpose, the study is divided into two parts; the direct and the inverse problems. In the direct problem, the radiative transfer equation is solved by the discrete transfer method from the knowledge of the absorption coefficient distribution and we obtain heat fluxes over the walls. Then the measured data are simulated virtually by adding the random errors to heat fluxes. The conjugate gradient method is used to solve the inverse problem to estimate the absorption coefficient distribution. As the measured data are less than the estimated parameters, a multi-step procedure is adopted to restrict the search region. Results show that the absorption coefficient distribution is well recovered in the medium with a low absorption coefficient by a two steps procedure. The results show that the location of inclusion may be found even by noisy data with 1% and 3% measurement errors. However, as the absorption coefficient increases, the location of inclusion is reconstructed in a three steps procedure and the inverse estimation becomes less efficient and time-consuming.

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1- Introduction

The applications of inverse radiation analysis in absorbing-emitting media include the prediction of temperature profiles or radiative properties by experimental radiation measurement. Such an inverse problem has applications in many fields; One of those is optical tomography, which tries to find out the location of a tumor. In this regard, the article aims to find the tumor location that its absorption coefficient is different from other medium backgrounds.

Many researchers have investigated the estimation of temperature field or source term distributions [1-6]. The inverse estimation of radiative properties has been reported by many researchers [7-13]. Many studies have reported on the simultaneous estimation of source term and radiative properties. Liu et al. [14, 15] and Li and Ozisik [16] investigated simultaneous estimation of temperature field and surface reflectivities. An inverse method for simultaneous estimation of temperature, absorption, and scattering coefficients was proposed by Zhou et al. [17, 18]. Zhou and Han [19] investigated an inverse reconstruction of temperature distribution, wall absorptivity, and absorption coefficient of a medium. Simultaneous determination of temperature distribution and soot volume fraction was reported by Lou and Zhou [20]. An inverse analysis for simultaneous estimation of temperature field and uniform medium properties for a two-dimensional furnace from the knowledge of exit intensity at boundary surfaces was proposed by Liu et al. [21]. Recently Hosseini Sarvari [22] presented an inverse analysis for estimation of absorption coefficient distribution in plane-parallel gray-diffuse media by the measurement of exit intensity received by radiation detectors at boundary surfaces. In the field of optical tomography, Klose and Hielscher [23] investigated an iterative reconstruction scheme for optical tomography, using the finite-difference discrete-ordinate formulation and the adjoint differentiation scheme to solve the equation of radiative transfer and the inverse problem, respectively. Comparison of diffusion approximation with Radiation Transfer (RT) analysis for light transport in tissues was investigated by Guo et al. [24]. The results show that RT modeling is more efficient and accurate. Kim and Charette [25] reported a frequency domain optical tomography using the conjugate gradient method without line search. Qiao et al. [26] investigated a reconstruction scheme for the fluorescence tomography based on the Time-Domain Radiative Transfer Equation (TD-RTE) by using an inverse method. Improved optical tomography based on a hybrid frequency-domain and time-domain radiative transfer model was presented by Zhao et al. [27].

In this article, we present an inverse analysis for estimation of absorption coefficient distribution in a two-dimensional semitransparent medium from the knowledge of heat fluxes over the walls. The inverse approach aims to develop a procedure to find the location of inclusion with a different absorption coefficient. The application of this approach is use-

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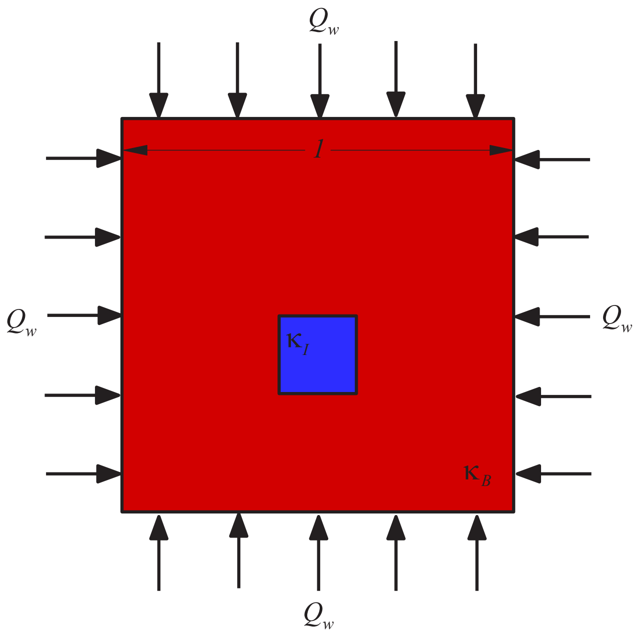


Fig. 1. A schematic specified radiative heat flux applied to the boundaries of a two-dimensional semitransparent medium (red-colored) with inclusion (blue-colored) in it with different radiative properties

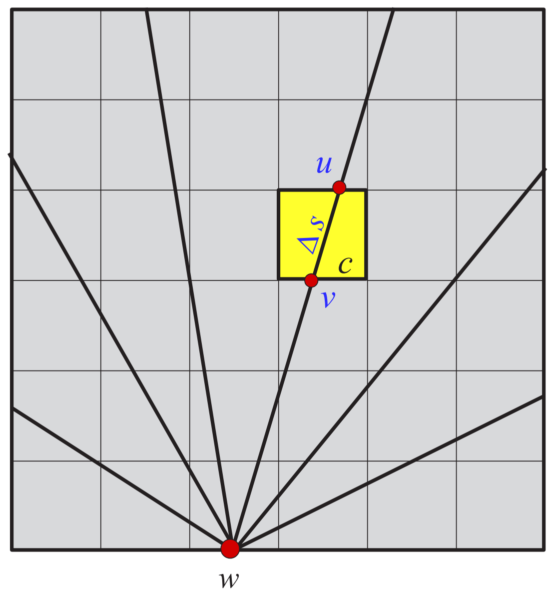


Fig. 2. A schematic semitransparent medium, spatial mesh, and ray trajectories from a surface element

ful in optical tomography with steady-state radiative media. The study is divided into two parts; the direct and the inverse problems. In the direct problem, the discrete transfer method [28] is used to solve the Radiative Transfer Equation (RTE). The inverse problem is solved by the Conjugate Gradient Method (CGM). A multi-step strategy is used to overcome the lack of measured data. Results show that the inverse estimation can successfully estimate the location of inclusion with a different absorption coefficient, especially for the media with a low absorption coefficient.

2- Description of Problem

Fig. 1 shows a simple physical model of a two-dimensional absorbing-emitting semitransparent medium. The walls are black with specified temperature and radiative properties. As shown in Fig. 1, the whole medium has the same specified absorption coefficient κ_B (red-colored medium), except for inclusion inside it, where its absorption coefficient, κ_I , is different from the background (blue-colored medium). Such a medium may be seen in tomography analysis, where the optical properties of tumors are different from the biological tissue. The inverse problem aims to find the location and magnitude of the absorption coefficient in inclusion, from the knowledge of radiative heat fluxes over the boundaries.

3- Direct Problem

The equation of radiative transfer in an absorbing-emitting medium is as follows [28]:

$$\frac{dI(s)}{ds} + \kappa I(s) = \kappa I_b \tag{1}$$

where the boundary conditions for black walls are given by

$$I(0) = I_{b,w} = \sigma T_w^4 / \pi \tag{2}$$

Here subscript w denotes the value at the wall surface. $I(s)$ is the radiation intensity, and κ is the medium's absorption coefficient. Radiative heat flux over the wall surface is obtained by [28]

$$Q = \pi I_{b,w} - \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_w(\theta,\phi) \cos\theta \sin\theta \, d\theta \, d\phi \tag{3}$$

To solve the RTE, the medium is discretized by a mesh with square cells. The discrete transfer method includes ray tracing along the rays emanating from surface elements. The center of each wall surface element is the source of radiative rays, which propagate along with the discrete solid angles and travel straight paths through the domain until reach other surface elements[29] (see Fig. 2).

Integrating Eq. (1) over the path lengths through the volume element leads to the following relation

$$I_c^v = I_c^u \exp(-\kappa_c \Delta s_c) + I_{b,c}[1 - \exp(-\kappa_c \Delta s_c)] \tag{4}$$

and the radiative heat flux at the wall surface is approximated by:

$$Q \approx \pi I_{b,\mathrm{w}} - \sum_{j=1}^{J} I_{w,j}\,\omega_j \tag{5}$$

where

$$\omega_j = \cos\theta_{w,j}\,\sin\Delta\theta_{w,j}\,\Delta\phi \tag{6}$$

is the weight associated with direction j. The numerical approach for the DTM is described in detail in Ref. [28] and will not be repeated.

4- Inverse Problem

One of the efficient methods for solving inverse problems is the Conjugate Gradient Method (CGM), which is used for linear and nonlinear inverse problems. In this study, the unknown parameter is the absorption coefficient. The CGM is the iterative procedure that is based on the minimization of the objective function, which is given by:

$$F(\vec{\kappa}) = [\vec{Q}_d - \vec{Q}_e(\vec{\kappa})]^T [\vec{Q}_d - \vec{Q}_e(\vec{\kappa})] \tag{7}$$

Here the $\vec{\mathbf{Q}}_\mathbf{d}$ and $\vec{\mathbf{Q}}_\mathbf{e}(\vec{\boldsymbol{\kappa}})$ are the vectors containing the measured and the estimated radiative heat fluxes, respectively. $\vec{\kappa}^k$ is the vector of independent absorption coefficients over the volume cells. Absorption coefficients are updated by:

$$\vec{\kappa}^{k+1} = \vec{\kappa}^k - \beta^k \vec{d}^k \tag{8}$$

where $\vec{d}^k$ is the direction of descent and β^k is the search step size, which is defined by:

$$\vec{d}^k = \nabla F(\vec{\kappa}^k) + \gamma^k \vec{d}^{k-1} \tag{9}$$

$$\beta^k = \frac{[\mathbf{S}^k \vec{d}^k]^T [\vec{Q}_d^k - \vec{Q}_e^k(\kappa)]}{[\mathbf{S}^k \vec{d}^k]^T [\mathbf{S}^k \vec{d}^k]} \tag{10}$$

In Eq. (9), γ^k is the conjugation coefficient which is calculated by:

$$\gamma^k = \frac{\sum_{n=1}^{N} [\nabla F_n(\vec{\kappa}^k)]^2}{\sum_{n=1}^{N} [\nabla F_n(\vec{\kappa}^{k-1})]^2} \quad , \; with \;\; \gamma^0 = 0 \tag{11}$$

Here, N is the number of unknown parameters. $\nabla F_n(\vec{\kappa}^k)$ is the n-th component of gradient direction, which is obtained by differentiation of the objective function, Eq. (7), with respect to $\vec{\kappa}^k$ as:

$$\nabla F(\vec{\kappa}^k) = -2(\mathbf{S}^k)^T [\vec{Q}_d^k - \vec{Q}_e^k(\vec{\kappa}^k)] \tag{12}$$

where $\mathbf{S}^k$ is the sensitivity matrix. The components of the sensitivity matrix are defined as

$$S_{mn}^k = \frac{\partial Q_{e,m}^k}{\partial \kappa_n^k} \;\; , \;\; m = 1,\dots,M \;\; , \;\; n = 1,\dots,N \tag{13}$$

where N and M are the numbers of unknown parameters and the number of surface elements, respectively. The stopping criterion for terminating the iterative procedure is

$$F(\vec{\kappa}^{k+1}) - F(\vec{\kappa}^k) < \xi \tag{14}$$

where ξ is a small positive number, say 10^{-20}.

5- Sensitivity Problem

Sensitivity coefficients, S_{mn}, expresses the sensitivity of wall heat flux, Q_m, with respect to the variation of κ_n. To obtain the sensitivity elements, the direct problem given by Eq. (5) is differentiated with respect to κ_n^k as:

$$S_{mn} = \frac{\partial Q_{e,m}^k}{\partial \kappa_n^k} = \sum_{j=1}^{J} \frac{\partial I_{w,j}^k}{\partial \kappa_n^k}\,\omega_j \tag{15}$$

where $\partial I_{w,j}^k / \partial \kappa_n^k$ is calculated by differentiation of Eq. (4) as:

$$\frac{\partial I_{w,j}^k}{\partial \kappa_n^k} = \frac{\partial I_c^u}{\partial \kappa_n^k} \exp(-\kappa_c \Delta s_c) + (I_{b,c} - I_c^u)\exp(-\kappa_c \Delta s_c)\,\delta_{cn} \tag{16}$$

where δ_{cn} is the Kronecker delta defined as:

$$\delta_{cn} = \begin{cases} 1 & when \;\; c = n \\ 0 & when \;\; c \neq n \end{cases} \tag{17}$$

6- Simulation of the Measured Data

In a real problem, the experimental data contain some errors due to measurement methods and instrumentation. To consider the effects of random errors in the measured data, normal distributed random errors are imposed on parameters

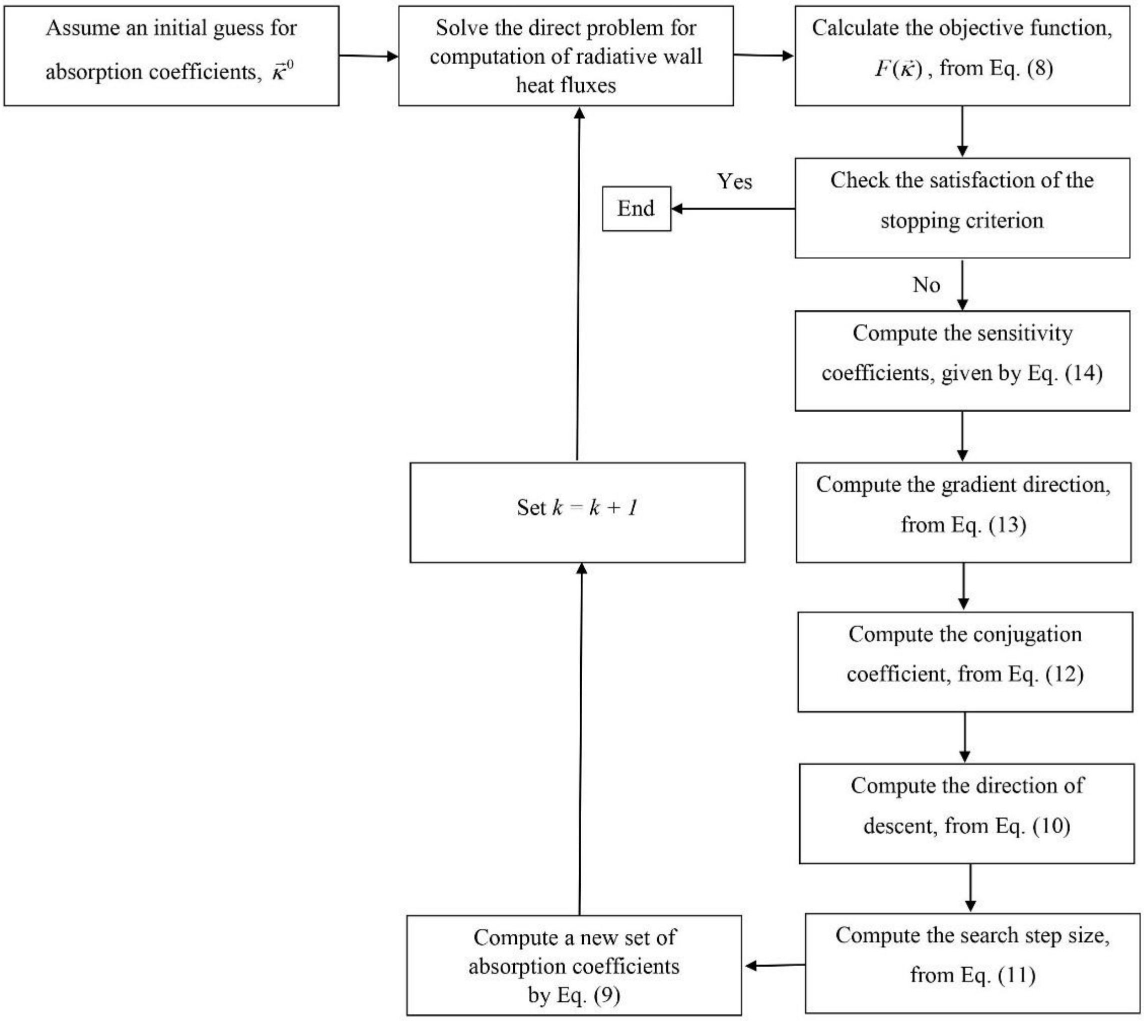


Fig. 3. Computation procedure for solving the inverse problem by the CGM

$$Q_{d,m} = Q_{ex,m} + \vartheta_m \zeta \tag{18}$$

where ζ s a normal distributed random error with zero mean and unit standard deviation, and ϑ_m is given by [22]:

$$\vartheta_m = \frac{Q_{ex} \times \eta}{2.576} \tag{19}$$

which means that 99% of normally distributed ϑ_m is within $\pm$2.576 standard deviation from the mean value [22]. Here, η is the measurement error associated with the measured data.

7- Computational Algorithm

The computational algorithm for solving the inverse problem by the CGM is summarized as follows:

1. Assume an initial guess for absorption coefficients, $\vec{\kappa}^0$.
2. Solve the direct problem with an available estimation of unknown parameters, and compute the radiative heat fluxes over the wall surface elements.
3. Calculate the objective function, $F(\vec{\kappa})$, from Eq. (7). Stop the iterative procedure if the stopping criterion is satisfied. Otherwise, go to step 4.
4. Compute the sensitivity coefficients, given by Eq. (13).
5. Compute the gradient direction, from Eq. (12), and then compute the conjugation coefficient from Eq. (11).
6. Compute the direction of descent from Eq. (9).
7. Compute the search step size from Eq. (10).
8. Compute a new set of absorption coefficients by Eq. Set k = k + 1 and return to step 2.

The flowchart of the computational algorithm is shown in Fig. 3.

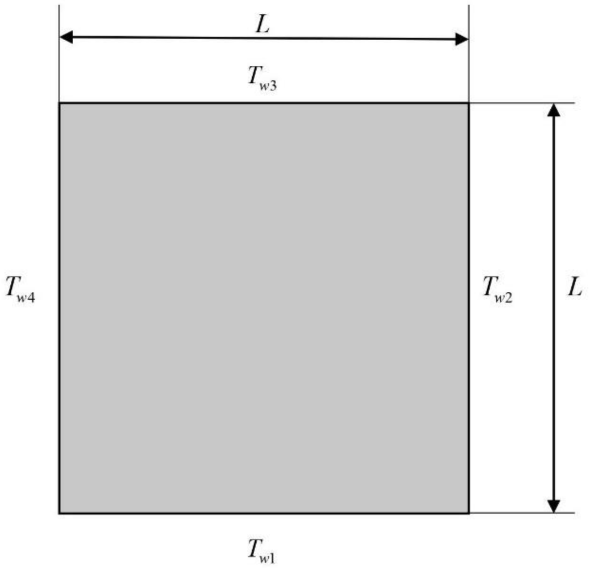


Fig. 4. Two-dimensional square absorbing-emitting medium in radiative equilibrium surrounded by black walls atTwi = 0 K , i= 2,3,4 and Tw1 = 1000 K

8- Verification of Direct Solution

The performance and accuracy of the direct solution with DTM are examined by comparing the results with those obtained by the Finite Element Method (FEM) benchmarks presented by Razzaque et al. [30]. Consider a square, absorbing-emitting medium in radiative equilibrium surrounded by black walls as depicted in Fig. 4. The optical thickness is $\kappa L = 1$, The walls are at $T_{wi} = 0$ K , $i = 2,3,4$ and $T_{w1} = 1000$ K .

The distribution of dimensionless heat flux, $Q/(\sigma T_{w1}^4)$, over the lower surface, as shown in Fig. 5, shows a good agreement with the benchmark.

9- Results and Discussion

The aim of the inverse problem is the reconstruction of the absorption coefficient distribution in a square enclosure with unit length as depicted in Fig. 1. We consider two test cases in two-dimensional absorbing-emitting media with low and high absorption coefficients, specified temperature distribution, and black walls. In all cases, two-dimensional 10×10 spatial and angular meshes are applied. The number of 40 heat fluxes over the center of control surfaces is considered as measured data. The exact location of inclusion is in the region where $0.4 \le x \le 0.6, 0.3 \le y \le 0.5$. All the specifications for both cases are listed in Table 1.

Test Case 1:This example aims to show the performance of the multi-step method to detect the location of inclusion in a participating medium from the knowledge of boundary heat fluxes. In this case, we solve a problem with a low absorption coefficient. The schematic of the absorption coefficient distribution is shown in Fig. 1. Now, the goal of the inverse problem is to reconstruct the absorption coefficient distribution. The absorption coefficient of background is considered to be $\kappa_B = 1\,\mathrm{m}^{-1}$. The inverse problem aims to detect the location of inclusion with $\kappa_I = 0.1\,\mathrm{m}^{-1}$.

As the number of measured data is less than the number of estimated results, the problem has multiple solutions. Hence, the inverse problem must be solved in a multi-step procedure. For the first step, a uniform κ -distribution is considered as the initial guess. Then the inverse procedure is adapted to find the distribution of the absorption coefficient. Since the number of measured data (the number of 40 heat fluxes for 40 control surfaces) is less than the number of volume elements (100 volume elements), at the first step, the estimated absorption coefficient distribution is far from the exact distribution. The result for the first step is shown in Fig. 6. As seen in Fig. 6, the distribution of κ is far from the exact solution. However, the result of the first step may predict the approximate

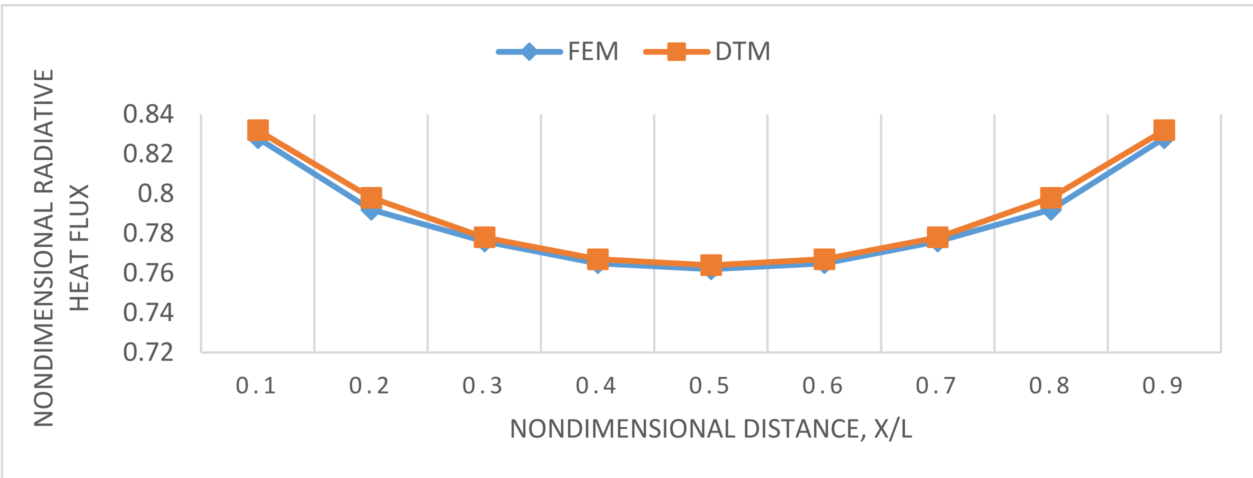


Fig. 5. Dimensionless heat flux distribution over the lower surface of the absorbing-emitting medium shown in Fig. 4

Table 1. Specifications of two test cases

Specification	Value
Length of the square enclosure, L	1 m
Number of angular meshes ($N_\phi \times N_\theta$)	10×10
Number of spatial meshes ($N_x \times N_y$)	10×10
Number of surface elements over the boundaries	40
The temperature of wall surfaces	20 K
x-Location of inclusion	$0.4\,\mathrm{m} \le x \le 0.6\,\mathrm{m}$
y-Location of inclusion	$0.3\,\mathrm{m} \le y \le 0.5\,\mathrm{m}$
Absorption coefficient for background and inclusion (κ_B , κ_I) for case 1	κ_B =1m^{-1}, κ_I =0.1m^{-1}
Absorption coefficient for background and inclusion (κ_B , κ_I) for case 2	κ_B =10m^{-1}, κ_I =5.0m^{-1}

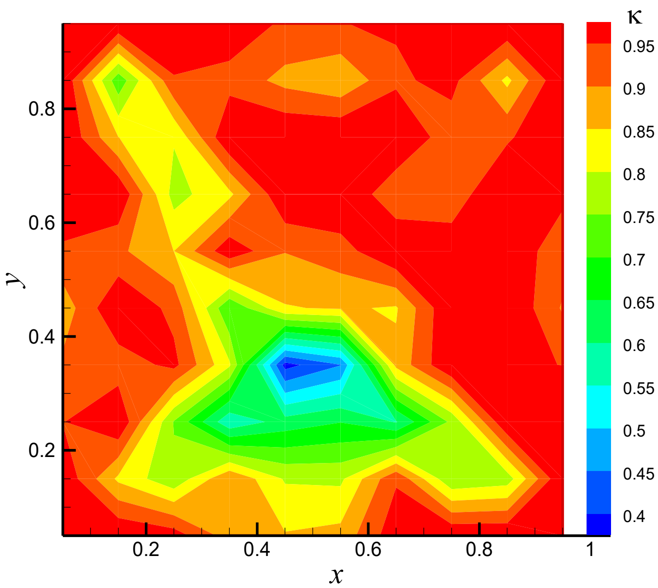
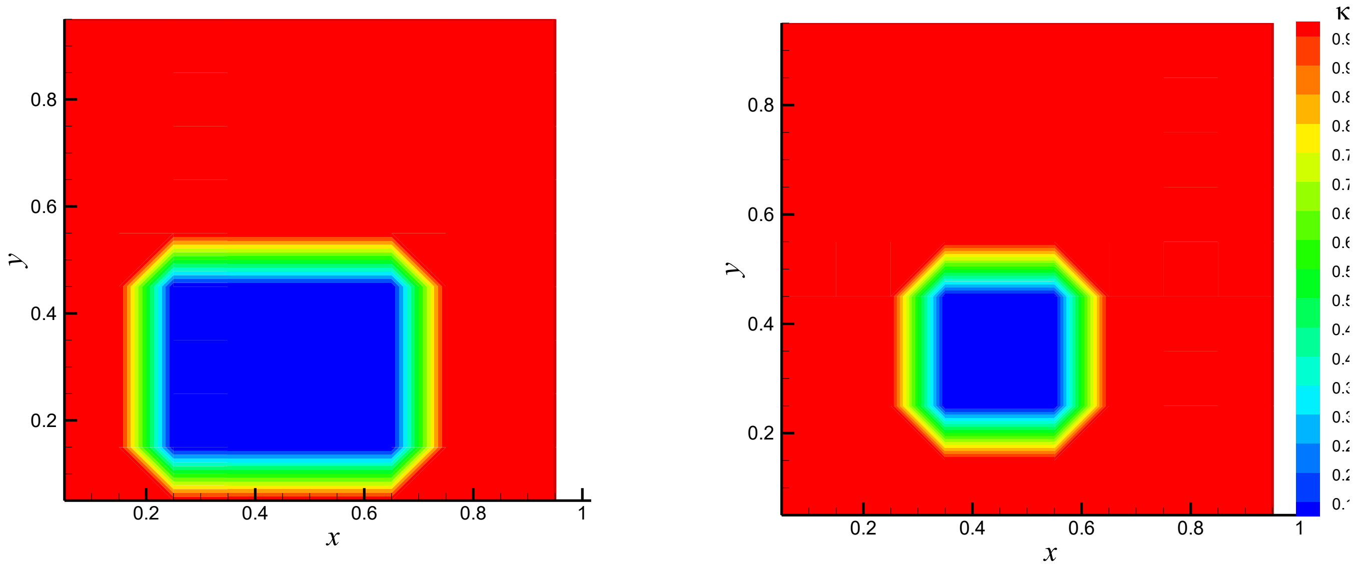


Fig. 6. Inverse reconstruction of the -distribution in an absorbing-emitting medium with a low absorption coefficient at the first step

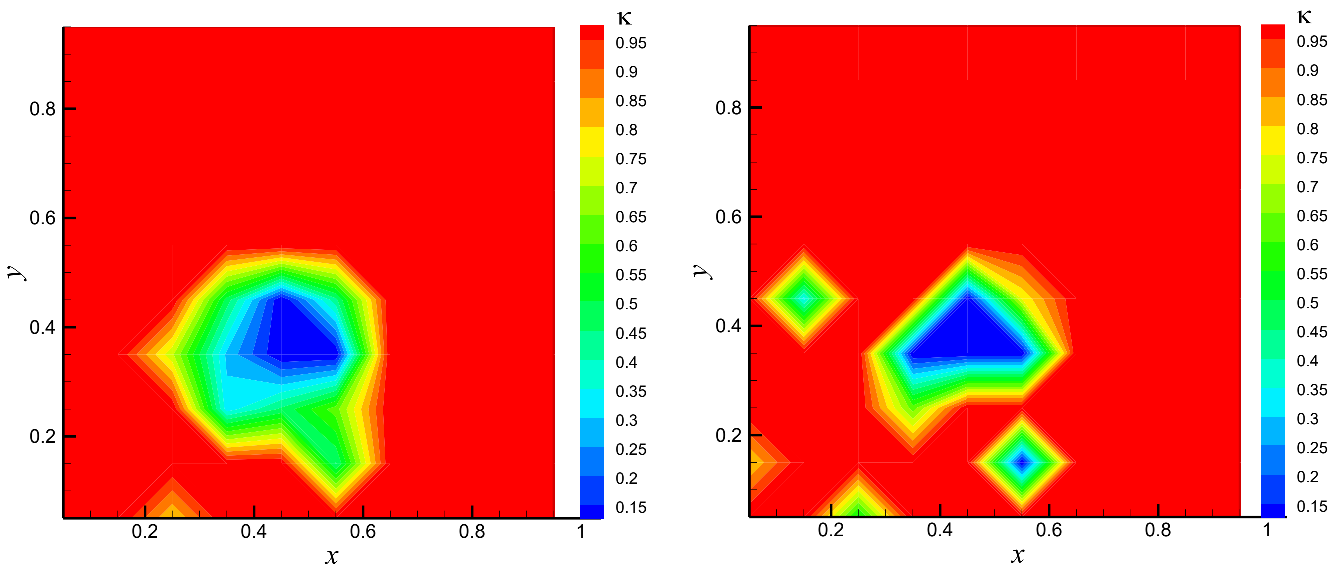
limits of the search region for the next step. Hence, for the second step, the search region is limited to the region with a different resolution with respect to the background. Therefore, according to the result of the first step, the search region is limited to the area where the inclusion is predicted to exist there, and the remaining surrounded region is considered as background with $\kappa = \kappa_B$. Then, for the second step, the inverse procedure is performed over the limited search region shown in Fig. 7(a). The result of the inverse reconstruction in the second step is shown in Fig. 7(b). As indicated, the estimated location of inclusion is well detected for the second step.

The results of the second step for the cases with 1% and 3% measurement errors are depicted in Figs. 8(a) and 8(b), respectively.The results show more deviation from the exact solution, however, the location of inclusion is constructed near the exact location.



(a) (b)

Fig.7. K -distribution (a) initial guess, and (b) inverse reconstruction in an absorbing-emitting medium with a low absorption coefficient at the second step



(a) (b)

Fig. 8. Inverse reconstruction of the -distribution with (a) 1% error, and (b) 3% error in an absorbing-emitting medium with a low absorption coefficient

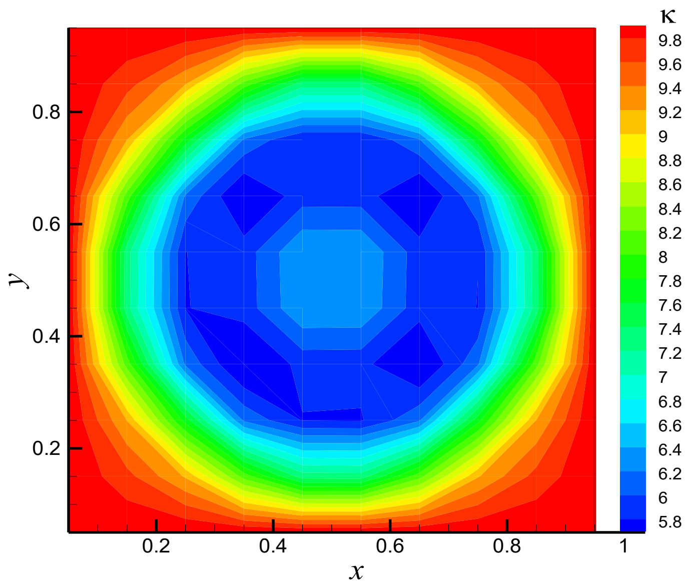
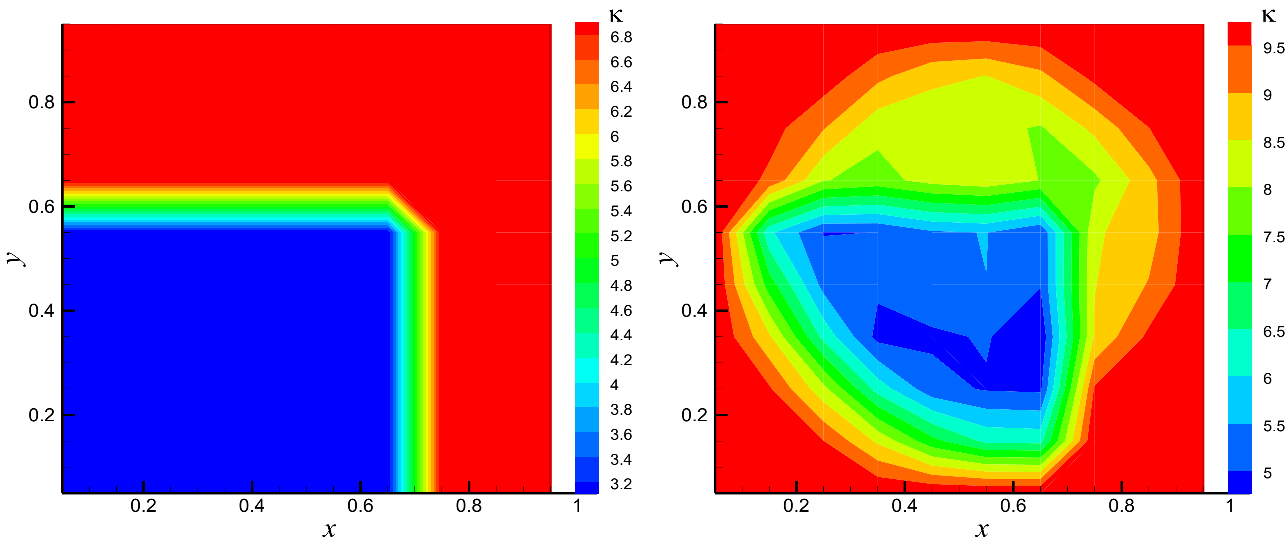


Fig. 9. Inverse reconstruction of the K -distribution in an absorbing-emitting medium with a high absorption coefficient at the first step

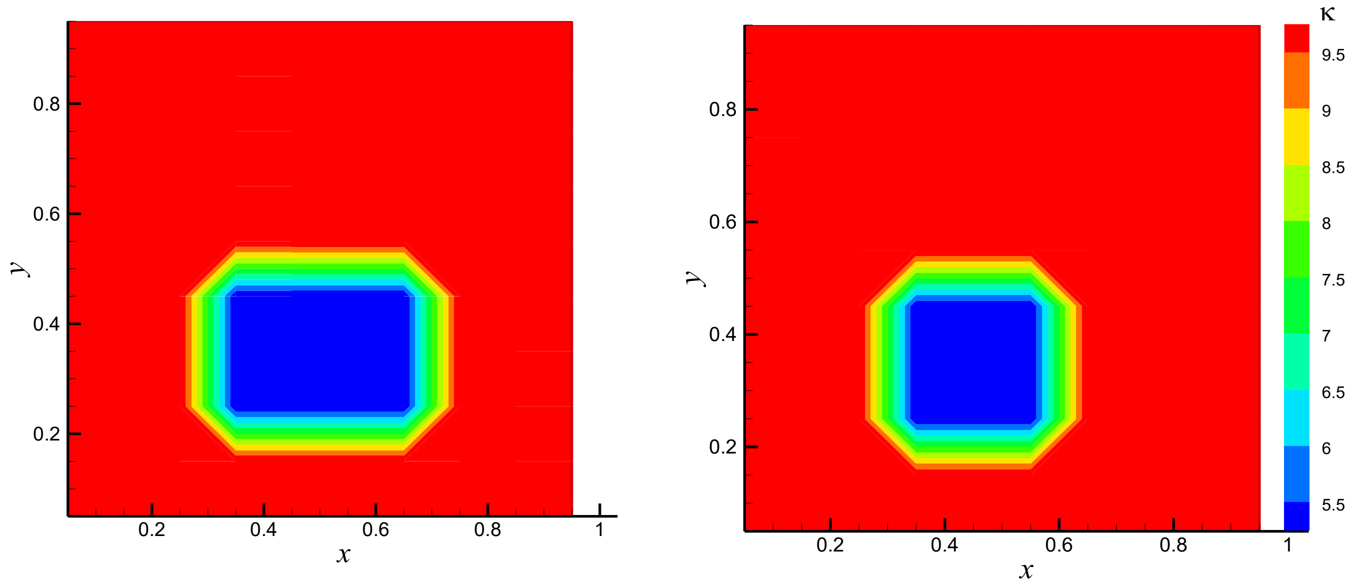


(a)

(b)

Figs. 10. -distribution (a) initial guess, and (b) inverse reconstruction in an absorbing-emitting medium with a high absorption coefficient at the second step

Test Case 2: In this case, we assume a thick medium with a high absorption coefficient. The absorption coefficient for the background and inclusion are considered to be $\kappa_B = 10\,\mathrm{m}^{-1}$ and $\kappa_I = 5\,\mathrm{m}^{-1}$, respectively. Like the previous case, we consider a uniform initial guess for the absorption coefficient. The result for the first step is shown in Fig. 9. As seen, the result of the first step is very far from the exact solution and worse than that for test case 1. This arises from the fact that for thick media the radiative energy is damped by the medium before it reaches the wall surfaces where the heat fluxes are measured. Hence, according to the result of the first step, we consider an alternative initial guess, shown in Fig. 10(a), and repeat the inverse procedure. The result of the second step is shown in Fig. 10(b). Although, the results of the second step show the existence of inclusion, the exact location of inclusion is not known yet. Hence, for the third step, we limited the search zone to a rectangular shown in Fig. 11(a). The result of inverse reconstruction is shown in Fig. 11(b). As shown, the location of inclusion is estimated exactly after three steps.



(a) (b)

Figs. 11. -distribution (a) initial guess, and (b) inverse reconstruction in an absorbing-emitting medium with a high absorption coefficient at the third step

10- Conclusion

An inverse approach was presented to estimate the location of inclusion with different optical properties in a two-dimensional absorbing-emitting medium. The radiative transfer equation was solved by the discrete transfer method, and the inverse problem to minimize the objective function was solved by the conjugate gradient method. The direct and inverse problems were linked together to reconstruct the absorption coefficient distribution. An effective approach was used to calculate the sensitivity coefficients by differentiation of governing equations with respect to the local absorption coefficient. In a square medium with 100 volume elements and 10 surface elements on each side, 40 measured heat fluxes over the boundary surface were used to recover the number of 100 discrete values of absorption coefficients over the volume elements. Since the number of measured data was less than the number of estimated absorption coefficients, a multi-step procedure was applied, where the first step aimed to restrict the search region into a smaller region within the medium. The results show that the inverse method is successful to reconstruct the location of inclusion for thin media, even with noisy data by applying 1% and 3% measurement errors. However, as the radiation problem tends to diffusion problem for thick media, the inverse method does not work as well as that for thin media.

Nomenclature

d	Direction of descent
F	Objective function
I	Intensity, W/m^2.sr
L	Square side size, m
M	Number of surface elements
N	Number of unknown parameters
Q	Heat flux, W/m^2
S	Sensitivity
s	Geometric path length, m
T	Temperature, K

Greek symbols

β	Search step size
γ	Conjugation coefficient
ζ	Normal distributed random error

η	Measurement error
θ	Polar angle, rad
κ	Absorption coefficient, m^{-1}
σ	Stefan-Boltzmann constant
ϕ	Azimuthal angle, rad
ω	Weight
ϑ_m	Standard deviation
Subscript	
b	Black body
B	Background
c	Cell
e	Estimated value
d	Measured value
ex	Exact value
I	Inclusion
j	Ray direction
m	Counter of surface elements
n	Counter of unknown parameters
w	Wall
Superscript	
*	Reference condition

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