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Nonlinear Control of Bilateral Teleoperators Interacting Non-passive Termination Forces

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ABSTRACT: In this paper, an interconnection and damping assignment passivity based controller is developed for nonlinear bilateral teleoperation system. The aim is to track the position and force in the teleoperation system in the presence of non-passive external interactions and asymmetric variable time-delay in the communication channel. For this end, a nonlinear control law is designed based on the notion of time-delay Port-Hamiltonian systems for unforced teleoperator and the Lyapunov-Krasovskii theorem. Sufficient synthesis conditions are derived in terms of linear matrix inequalities to tune the parameters of controller. Then, by Lyapunov redesign scheme, an auxiliary controller is developed to assure the stable position tracking in the presence of non-passive operator and/or environment. The main contribution of the proposed method is that the stability and position tracking of system is attained via a fixed-structure controller in the presence of non-passive interaction forces without need to their dynamical models and force sensor. Since the proposed design conditions include the upper bounds of the varying time-delays and their rates; they are less conservative than some rival methods in literature. Finally, transparency of the proposed scheme is proved. Simulation results on a 2-degree of freedom teleoperation system are compared to rival methods to demonstrate the merits of the proposed strategy.

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1. INTRODUCTION

In bilateral teleoperation systems, one remote robot follows the motions of a local one. Generally, a human operator manipulates the local robot, the resulting movement is communicated to the remote manipulator which imitates that motion; and then the remote robot returns back reaction forces of the environment to the local robot to give the operator a feeling of telepresence [1,2]. An important challenge in design of bilateral teleoperation systems is to provide the stability, position and force tracking by the controller especially in the presence of time-delay in the communication channel. Compensation of destructive effects of time-delay in the controller design for teleoperation systems has been studied by many researchers in the fields of robotics and control during the recent years [3].

A survey by Nuno et al. [4] categorizes the telemanipulator controllers into three groups: scattering based, damping injection and adaptive schemes. The aim of the scattering based controllers is to render the connection channel passive by imitating the action of an electrical lossless transmission line. These schemes can provide stability independent of delay value but cannot guarantee position tracking, as originally designed [5,6]. In the second group the Proportional (P) or Proportional Derivative (PD) controllers with damping (d) injection term are considered [7-9], in which the damping injection by passive output feedback ensures asymptotic stability. These methods result in position tracking with delay-dependent stability. In general damping injection schemes create a slow response. Eventually, the adaptive strategies [10,11] lead to position tracking in spite of constant delay, through estimation of the parameters of the teleoperation system. Only scattering based adaptive approaches [12] can stabilize the teleoperation system with variable time-delay.

The stability of teleoperation system in the most of aforementioned controllers is analyzed by Lyapunov-Krasovskii (LK) argument under the restrictive assumption that the environment and human operator are energetically passive. Passive interaction forces decrease the robots' velocities which benefit the system stability, but this property may be violated in many real-world applications. For instance, the human operator has non-passive behavior in the rigid grasping [13] or trajectory following tasks [14] or the environment is non-passive when external forces are applied to the teleoperator, in situations such as mining, drilling or beating heart surgery [15]. In general, the non-passive behavior of human or environment can be modeled by a constant force which may deteriorate the performance of the system or even destabilize it. Only few papers have considered this issue in the controller design and stability analysis of teleoperation systems. These papers can be classified in to two groups. In the first group [16,17] the interaction forces are compensated locally using sensors for measurement

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of the interaction forces, which are costly and noisy. In the second group [18-22], force sensors are not required and the controller parameters are tuned to retain the stability of system interacting with non-passive termination.

In reference [18], the pioneer work addressing this issue, the teleoperation system interacting with constant human force and passive environment in the presence of constant communication delay is stabilized using LK theorem. In reference [19], linear teleoperator with non-passive human operator and environment is controlled by common PD+d approach neglecting the communication delay. In reference [20], a teleoperation system interacting with constant human force and passive environment and varying communication delay, controlled by PD+d scheme is studied, in which using LK theorem the asymptotical stability of the closed-loop system is guaranteed. In references [18,23], the LK theorem is used for stability analysis of a nonlinear teleoperator controlled by P+d based scheme, where the system is subject to varying communication delay, non-passive operator and environment. In references [21,22], the nonlinear autonomous teleoperator in the presence of varying time-delay is considered. Using LK theorem delay-dependent criteria are derived to determine controller gains provided that the non-passive interaction forces satisfy hard-to-hold conditions which relate their values to current amounts of positions and velocities of local and remote robots.

Interconnection and Damping Assignment Passivity Based Controller (IDA-PBC) is a nonlinear state-feedback control law that is designed to tackle stabilization and tracking problems of physical systems by assigning a desired Port Hamiltonian (PH) structure to the closed-loop [24]. PH description is a natural method of formulating a physical system in terms of its energy interchange with the environment through its ports. The Hamiltonian function in a PH model is considered as the sum of kinetic and potential energies in the physical systems, and can be used as an appropriate candidate of Lyapunov function in stability analysis. When time-delay appears in PH model, suitable LK functional is constructed using Hamiltonian function of system for stability analysis [25-27].

In this paper, the notion of IDA-PBC is adopted to design controller for bilateral teleoperator in the presence of asymmetric variable time-delay without any restriction on dynamics and passivity of interacting terminators. In the first step, the structure of IDA-PBC is designed for the teleoperation system without considering interacting forces. Then, an appropriate LK functional composed of Hamiltonian function of system is employed to derive Linear Matrix Inequality (LMI) conditions to tune the parameters of IDA-PBC such that the stability of motion is achieved for unforced system. In the second step, in the Lyapunov redesign framework, an auxiliary controller is developed to assure the stability of the system in spite of non-passive human and environment.

The contributions of this paper are threefold: First, the theory of IDA-PBC is extended to time-delay systems. Since, the proposed synthesis conditions include the upper bounds of varying time-delays and their rates; they are less conservative than some rival methods in the literature. Second, no restrictive assumptions are made on the passivity and dynamics of interacting forces. Third, force sensors are not required to implement the proposed control strategy. The comparative simulation results are presented to show the merits of the proposed method.

It is worth noting that, the proposed P+d based controller in reference [21] includes some adaptive terms to compensate the effects of external forces. Therefore, the controller preserves stability and tracking using computationally demanding adaptive terms. Moreover, non-passive part of the operator force is modeled by an impedance with negative spring-damper, which is only a special case of general non-passive model (i.e., unknown force with unknown dynamic model), and the environment non-passive forces were estimated. While in our work, the interaction forces are considered in more general case (i.e. unknown forces with unknown dynamic models) and the fixed structure controller, designed using Lyapunov redesign method, maintains stability and performance objectives without any online adaptive part. The considered teleoperation system in reference [19] is linear and time-invariant. Moreover, the time-delay in communication channel is neglected. Proportional-derivative based controllers are considered for system, which need force sensors for implementation. In our work, the nonlinear model of teleoperation system is considered in the presence of asymmetric varying delay in communication channel. Furthermore, the implementation of the developed controller requires no force sensors.

The organization of the paper is as follows: in Section 2, the general dynamical model of the considered teleoperator is presented; in Section 3, the proposed IDA-PBC controller for teleoperation system is developed and delay-dependent stability condition is obtained by LK theorem. Simulation results are presented in Section 4. Finally, conclusions are given in Section 5.

2. PROBLEM FORMULATION

In this section, the Euler-Lagrange equations of the considered nonlinear teleoperation system comprising n-Degrees Of Freedom (DOF) manipulators are recalled from reference [7]. Then, the relations are reformulated as an affine state space model.

2-1- Euler-Lagrange model of system

The nonlinear equations of local and remote manipulators together with the interaction forces are as follows [7]:

$$\begin{aligned} M_l(q_l)\ddot{q}_l + C_l(q_l,\dot{q}_l)\dot{q}_l + g_l(q_l) &= \tau_l^* - \tau_h \\ M_r(q_r)\ddot{q}_r + C_r(q_r,\dot{q}_r)\dot{q}_r + g_r(q_r) &= \tau_e - \tau_r^* \end{aligned} \quad (1)$$

where $q_i, \dot{q}_i, \ddot{q}_i \in \mathbb{R}^n$ are the joint positions, velocities and accelerations, $M_i(q_i) \in \mathbb{R}^{n\times n}$ are the inertia matrices and

$C_i(q_i,\dot{q}_i)\in\mathbb{R}^{n\times n}$ are the Coriolis and centrifugal effects, $g_i(q_i)\in\mathbb{R}^n$ are the gravitational force vectors, $\tau_i^*\in\mathbb{R}^n$ are applied control forces and $\tau_h,\tau_e\in\mathbb{R}^n$ represent the external forces exerted by the human operator and environment to the local and remote manipulators, respectively. Here, demonstrate the local manipulator and $i=r$ the remote one.

The dynamical model of manipulators presented in Eq. (1) have the following property [7]:

P1. The inertia matrix $M_i(q_i)$ of a robot is bounded as $0<\lambda_m I\le M_i(q_i)\le\lambda_M I<\infty$.

To simplify the computation, the gravitational forces are locally pre-compensated (i.e., $\tau_l^*=\tau_l+g_l(q_l)$ and $\tau_r^*=\tau_r-g_r(q_r)$); so, the dynamical model Eq. (1) is changed to:

$$\begin{aligned}M_l(q_l)\ddot{q}_l+C_l(q_l,\dot{q}_l)\dot{q}_l&=\tau_l-\tau_h\\ M_r(q_r)\ddot{q}_r+C_r(q_r,\dot{q}_r)\dot{q}_r&=\tau_e-\tau_r\end{aligned}\tag{2}$$

It is assumed that the local and remote robots exchange data by a communication medium which imposes variable time-delays, $T_i(t)$ that have known upper bounds h_i, i.e., $0\le T_i(t)\le h_i<\infty$ and do not grow or decrease faster than a known value μ_i, i.e., $|\dot{T}_i|<\mu_i; i=l,r$.

2-2- State space model

By definition of state vector $x_l=\begin{bmatrix}q_l\\ \dot{q}_l\end{bmatrix}:=\begin{bmatrix}x_{1l}\\ x_{2l}\end{bmatrix}\in\mathbb{R}^{2n}$, the input vectors $u_l=\tau_l$ and $w_l=\tau_h$, the affine model for local manipulator is achieved as

$$\dot{x}_l=f_l(x_l)+g_{ul}(x_l)u_l+g_{wl}(x_l)w_l\tag{3}$$

where $f_l(x_l)$ describes the sub-system dynamics, $g_{ul}(x_l)$ and $g_{wl}(x_l)$ are the control and external input functions

$$f_l(x_l)=\begin{bmatrix}x_{2l}\\ -M_l^{-1}(x_{1l})C_l(x_{1l},x_{2l})x_{2l}\end{bmatrix},$$

$$g_{ul}(x_l)=\begin{bmatrix}\underline{0}\\ M_l^{-1}(x_{1l})\end{bmatrix};g_{wl}(x_l)=\begin{bmatrix}\underline{0}\\ -M_l^{-1}(x_{1l})\end{bmatrix};\underline{0}\in\mathbb{R}^{n\times n}$$

Similarly, the model of remote manipulator is

$$\dot{x}_r=f_r(x_r)+g_{ur}(x_r)u_r+g_{wr}(x_r)w_r\tag{4}$$

where $x_r=\begin{bmatrix}q_r\\ \dot{q}_r\end{bmatrix}:=\begin{bmatrix}x_{1r}\\ x_{2r}\end{bmatrix}\in\mathbb{R}^{2n}$, $u_r=\tau_r, w_r=\tau_e$ and

$$f_r(x_r)=\begin{bmatrix}x_{2r}\\ -M_r^{-1}(x_{1r})C_r(x_{1r},x_{2r})x_{2r}\end{bmatrix},$$

$$g_{ur}(x_r)=\begin{bmatrix}\underline{0}\\ -M_r^{-1}(x_{1r})\end{bmatrix};g_{wr}(x_r)=\begin{bmatrix}\underline{0}\\ M_r^{-1}(x_{1r})\end{bmatrix};\underline{0}\in\mathbb{R}^{n\times n}$$

By augmenting the states of the local and remote manipulators in the vector $x\in\mathbb{R}^{4n}$, the overall system can be expressed as

$$\dot{x}=f(x)+g(x)(u-w)\tag{5}$$

where

$x=\left[x_l^T,x_r^T\right]^T$, $u=\left[u_l^T,u_r^T\right]^T$, $w=\left[w_l^T,w_r^T\right]^T$,

$f(x)=\left[f_l(x_l)^T,f_r(x_r)^T\right]^T$ and

$$g(x)=\begin{bmatrix}g_{ul}&\underline{0}\\ \underline{0}&g_{ur}\end{bmatrix};\underline{0}\in\mathbb{R}^{2n\times n}.$$

It should be noted that the input vector u is calculated and applied by the controller and the input w is interaction input caused from human/environment. The problem of interest is to determine the control input u for both of manipulators to achieve stable position coordination in spite of varying time-delay in data exchange between them without any limiting assumptions on w.

3- MAIN RESULTS

The schematic of the considered teleoperation system is depicted in Fig. 1. In general, the control signal $u=\left[u_l^T,u_r^T\right]^T$ is constructed from two terms as $u=u_{ida}-v$, wherein u_{ida} is responsible for desirable performance in unforced system and the role of v is to guarantee stable position coordination despite any interaction forces which prevent the IDA controller from providing stability in the system.

3-1- IDA-PBC design

Consider the local manipulator Eq. (3) when $w_l=0$. Algebraic IDA-PBC strategy in which the desired Hamiltonian function is defined for the closed-loop system and the controller structure is determined by solving a matching condition is employed for synthesis of IDA-PBC for the unforced manipulator. The aim is to find a control torque u_l such that the resulting closed-loop for local subsystem, Eq. (3) be as

$$\dot{x}=F_l\,\nabla_{x_l}H_l\left(x_l,x_{1r}(t-T_r)\right)\tag{6}$$

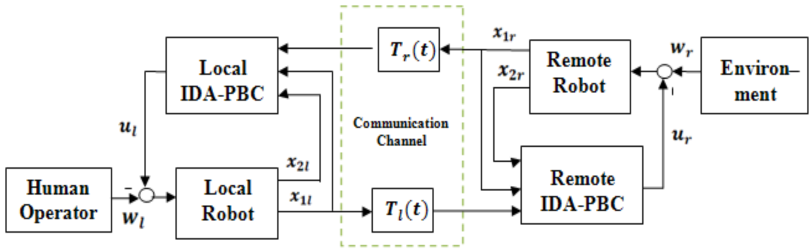


Fig. 1. Schematic of teleoperator with the proposed controllers

where the desired closed-loop Hamiltonian function is chosen to be as

$$H_l\left(x_l, x_{1r}\left(t-T_r\right)\right)=\frac{1}{2} x_{2l}{}^T B_l x_{2l}+ \frac{1}{4}\left(x_{1l}-x_{1r}\left(t-T_r\right)\right)^T K_l\left(x_{1l}-x_{1r}\left(t-T_r\right)\right) \tag{7}$$

in which the first term indicates desired kinetic energy of the local manipulator and the second one represents control energy. The constant matrices B_l, K_l and $F_l=\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \in \mathbb{R}^{2n\times 2n}$ are determined later. By considering the full-rank left annihilator matrix of g_{ul} as $g_{ul}^{\perp}(x_l)=[\underline{I} \quad \underline{0}]$; $\underline{0}, \underline{I} \in \mathbb{R}^{n\times n}$, the matching equation is

$$g_{ul}^{\perp}(x_l)\left(F_l \nabla_{x_l} H_l\left(x_l, x_r\left(t-T_r\right)\right)-f_l(x_l)\right)=0 \tag{8}$$

which can be trivially solved by $F_l=\begin{bmatrix} \underline{0} & B_l^{-1} \\ F_{21} & F_{22} \end{bmatrix}$. So, the control law is as the following

$$u_l =: u_{l_{ida}}=\left(g_{ul}^T\left(x_l\right) g_{ul}\left(x_l\right)\right)^{-1} g_{ul}^T\left(x_l\right) \left(F_l \nabla_{x_l} H_l\left(x_l, x_r\left(t-T_r\right)\right)-f_l\left(x_l\right)\right) \tag{9}$$

Similarly, for remote subsystem in Eq. (4) with $w_r=0$ we have

$$u_r =: u_{r_{ida}}=\left(g_{ur}^T\left(x_r\right) g_{ur}\left(x_r\right)\right)^{-1} g_{ur}^T\left(x_r\right) \left(F_r \nabla_{x_r} H_r\left(x_l\left(t-T_l\right), x_r\right)-f_r\left(x_r\right)\right) \tag{10}$$

where $F_r=\begin{bmatrix} \underline{0} & B_r^{-1} \\ F_{43} & F_{44} \end{bmatrix}$ and

$$H_r\left(x_{1l}\left(t-T_l\right), x_r\right)=\frac{1}{2} x_{2r}{}^T B_r x_{2r}+ \frac{1}{4}\left(x_{1l}\left(t-T_l\right)-x_{1r}\right)^T K_r\left(x_{1l}\left(t-T_l\right)-x_{1r}\right) \tag{11}$$

The unknown parameter of controllers F_i, B_i and K_i are chosen such that stability and desirable performance in position tracking and transparency are achieved for the unforced subsystems.

In what follows, computationally amenable LMI conditions are derived to tune efficiently the free parameters of the control laws. Note that the overall dynamic of the closed-loop system with the IDA controller can be expressed as a delayed PH model

$$\dot{x}=F_d \nabla_x H_d\left(x, \tilde{x}_l, \tilde{x}_r\right) \tag{12}$$

where

$$x=\left[x_l{}^T, x_r{}^T\right]^T,\ \tilde{x}_l=x\left(t-T_l\right),\ \tilde{x}_r=x\left(t-T_r\right),$$
$$H_d\left(x, \tilde{x}_l, \tilde{x}_r\right)=H_l\left(x_l, x_{1r}\left(t-T_r\right)\right)+ H_r\left(x_{1l}\left(t-T_l\right), x_r\right) \text{ and } F_d=\operatorname{diag}\left\{F_l, F_r\right\}.$$

Along the lines of reference [25], stability criteria are extracted for PH system with two variable time-delays to cope with the model of teleoperation system (Eq. (12)).

As the Hamiltonian function H_d is regular positive definite with regards to x. Then $\nabla_x H_d(x, \tilde{x}_l, \tilde{x}_r), \nabla_{\tilde{x}_l} H_d(x, \tilde{x}_l, \tilde{x}_r)$ and $\nabla_{\tilde{x}_r} H_d(x, \tilde{x}_l, \tilde{x}_r)$ can be written as

$$\nabla_x H_d\left(x, \tilde{x}_l, \tilde{x}_r\right)=F_1\begin{bmatrix} x \\ \tilde{x}_l \\ \tilde{x}_r \end{bmatrix}, \nabla_{\tilde{x}_l} H_d\left(x, \tilde{x}_l, \tilde{x}_r\right)= F_2\begin{bmatrix} x \\ \tilde{x}_l \\ \tilde{x}_r \end{bmatrix}, \nabla_{\tilde{x}_r} H_d\left(x, \tilde{x}_l, \tilde{x}_r\right)=F_3\begin{bmatrix} x \\ \tilde{x}_l \\ \tilde{x}_r \end{bmatrix} \tag{13}$$

where $F_1, F_2, F_3 \in \mathbb{R}^{4n\times 12n}$. The following theorem summarizes the stable position coordination condition of the closed loop teleoperation system (Eq. (12)).

Theorem1. The system (eq. (12)) is locally asymptotically stable provided that for the regular positive definite Hamiltonian function $H_d\left(x, \tilde{x}_l, \tilde{x}_r\right)$

1) there exist constant matrices L, S, Z_1, Z_2 with appropriate dimensions satisfying

$$
\begin{aligned}
&F_d^T + F_d \le L; \\
&F_d^T F_d \le S\ ; \\
&F_2^T F_2 \le F_1^T Z_1 F_1; \\
&F_3^T F_3 \le F_1^T Z_2 F_1;
\end{aligned}
\tag{14}
$$

2) There exist constant positive definite matrices P, Q_1, Q_2, R_1, R_2 with appropriate dimensions such that

$$
\Gamma := \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} \\ \Gamma_{12}^T & \Gamma_{22} & \underline{0} \\ \Gamma_{13}^T & \underline{0} & \Gamma_{33} \end{bmatrix} < 0 \tag{15}
$$

where $\underline{0} \in \mathbb{R}^{4n \times 4n}$ and

$$
\begin{aligned}
&\Gamma_{11} = L + Z_1 + Z_2 + PAF_d + \left(PAF_d\right)^T \\
&+Q_1 + Q_2 + h_l^2 R_1 + h_r^2 R_2; \\
&\Gamma_{12} = PBF_d; \\
&\Gamma_{13} = PCF_d; \\
&\Gamma_{22} = -\left(1-\mu_l\right) Q_1 + S; \\
&\Gamma_{33} = -\left(1-\mu_r\right) Q_2 + S;
\end{aligned}
$$

in which,

$$
\begin{aligned}
A &:= \nabla_{xx} H_d\left(x, \tilde{x}_l, \tilde{x}_r\right) = \frac{\partial \nabla_x H\left(x, \tilde{x}_l, \tilde{x}_r\right)}{\partial x}, \\
B &:= \nabla_{x\tilde{x}_l} H_d\left(x, \tilde{x}_l, \tilde{x}_r\right) = \frac{\partial \nabla_x H_d\left(x, \tilde{x}_l, \tilde{x}_r\right)}{\partial \tilde{x}_l} \\
C &:= \nabla_{x\tilde{x}_r} H_d\left(x, \tilde{x}_l, \tilde{x}_r\right) = \frac{\partial \nabla_x H_d\left(x, \tilde{x}_l, \tilde{x}_r\right)}{\partial \tilde{x}_r}.
\end{aligned}
$$

Proof. See the Appendix A.

3-2- Lyapunov redesign

Now, consider the teleoperation system with external forces, i.e. system Eq. (5) with $w \neq 0$. If the controller is chosen as

$$
u = u_{ida} - v \tag{16}
$$

where $u_{ida} = \left[u_{l_{ida}}{}^T, u_{r_{ida}}{}^T\right]^T$, regarding Eq. (12), the closed-loop system can be expressed as

$$
\dot{x} = F_d \nabla_x H_d\left(x, \tilde{x}_l, \tilde{x}_r\right) - g(x)v - g(x)w \tag{17}
$$

In theorem 2, v is computed to achieve stable position coordination in the teleoperation system despite bounded interaction forces.

Theorem 2. Consider the bilateral teleoperation system (Eq. (17)). The system is asymptotically stable and has coordinated positions if the control term v in Eq. (16) be as

$$
v := v\left(x, \tilde{x}_l, \tilde{x}_r\right) = -\eta \frac{\omega_1}{\|\omega\|} \tag{18}
$$

where $\omega = \left[\omega_1, \omega_2, \omega_3\right]^T$ with

$$
\begin{aligned}
\omega_1 &= -2\left(\nabla_x^T H_d\left(x, \tilde{x}_l, \tilde{x}_r\right)\left(I + PA\right) g(x)\right)^T \\
\omega_2 &= -2\left(\left(\nabla_{\tilde{x}_l}^T H_d\left(x, \tilde{x}_l, \tilde{x}_r\right) + \nabla_x^T H_d\left(x, \tilde{x}_l, \tilde{x}_r\right) PB\right) g\left(\tilde{x}_l\right)\right)^T \\
\omega_3 &= -2\left(\left(\nabla_{\tilde{x}_r}^T H_d\left(x, \tilde{x}_l, \tilde{x}_r\right) + \nabla_x^T H_d\left(x, \tilde{x}_l, \tilde{x}_r\right) PC\right) g\left(\tilde{x}_r\right)\right)^T
\end{aligned}
$$

and $\eta \ge \rho$, where $\begin{aligned}&\|W_w\| \le \rho, W_w = \left[w^T, \tilde{w}_l^T, \tilde{w}_r^T\right]^T, \\ &\tilde{w}_l := w\left(t - T_l\right),\ \tilde{w}_r := w\left(t - T_r\right)\end{aligned}$.

Proof. See the Appendix A.

Remark 1. The IDA controllers (Eqs. (9) and (10)) can provide input-to-state stability without need to Eq. (18), if the external forces satisfy the following condition

$$
W_w < -\frac{\lambda_{min}(\Gamma)\|\nabla H\|^2}{\|\omega\|} \tag{19}
$$

Where

$$
\nabla H = \begin{bmatrix} \nabla_x H_d\left(x, \tilde{x}_l, \tilde{x}_r\right)^T, \\ \nabla_{\tilde{x}_l} H_d\left(\tilde{x}_l, \tilde{\tilde{x}}_l, \tilde{\tilde{x}}_{rl}\right)^T, \\ \nabla_{\tilde{x}_r} H_d\left(\tilde{x}_r, \tilde{\tilde{x}}_{rl}, \tilde{\tilde{x}}_r\right)^T \end{bmatrix}^T, \tilde{\tilde{x}}_l := x(t - 2T_l),
$$

$\tilde{\tilde{x}}_r := x(t - 2T_r)$, $\tilde{\tilde{x}}_{rl} := x(t - T_l - T_r)$,, and $\lambda_{min}(\Gamma)$ is the smallest eigenvalue of the matrix Γ.

Remark 2. Since the parameters of the manipulators are difficult to be determined precisely in practice, often there are uncertainties in Eqs. (1) and (2). These dynamical uncertainties can be modeled by additive terms to the nominal model of system as

$$
\begin{aligned}
M_i(\mathrm{q}_i) &= \bar{M}_i(\mathrm{q}_i) + \Delta M_i(\mathrm{q}_i) \\
C_i\left(q_i, \dot{q}_i\right) &= \bar{C}_i\left(q_i, \dot{q}_i\right) + \Delta C_i\left(q_i, \dot{q}_i\right) \\
g_i(\mathrm{q}_i) &= \bar{g}_i(\mathrm{q}_i) + \Delta g_i(\mathrm{q}_i)
\end{aligned}
\tag{20}
$$

where $\bar{M}_i(\mathrm{q}_i), \bar{C}_i\left(q_i, \dot{q}_i\right)$ and $\bar{g}_i(\mathrm{q}_i)$ are the nominal model matrices/vectors and $\Delta M_i(\mathrm{q}_i), \Delta C_i\left(q_i, \dot{q}_i\right), \Delta g_i(\mathrm{q}_i)$ are norm bounded dynamical uncertainty matrices/vectors as follows:

$$
\|\Delta M_i(\mathrm{q}_i)\| \le \delta_M; \|\Delta C_i\left(q_i, \dot{q}_i\right)\| \le \delta_C; \|\Delta g_i(\mathrm{q}_i)\| \le \delta_g \tag{21}
$$

If the models of robots have uncertainty as Eq. (20) and the controller is designed using nominal model, the closed loop system is as the following

$$\dot{x} = \tilde{F}_d \nabla_x H_d\left(x, \tilde{x}_l, \tilde{x}_r\right) + g\left(x\right)\left(-w\right) \tag{22}$$

where $\tilde{F}_d = \mathrm{diag}\left\{\tilde{F}_l, \tilde{F}_r\right\}, \tilde{\mathrm{w}} = \left[(w_l + \Delta \mathrm{g}_l)^T \quad (w_r + \Delta \mathrm{g}_r)^T\right]^T$ and

$$\tilde{F}_l = \begin{bmatrix} 0 & B_l^{-1} \\ \alpha_l & \gamma_l \end{bmatrix}; \alpha_l = M_l^{-1}(\mathrm{q}_l)\bar{M}_l(\mathrm{q}_l)F_{21};$$
$$\gamma_l = M_l^{-1}(\mathrm{q}_l)\left(\bar{M}_l(\mathrm{q}_l)\right)F_{21} - \Delta C_l\left(q_l, \dot{q}_l\right)B_l^{-1}$$

$$\tilde{F}_r = \begin{bmatrix} 0 & B_r^{-1} \\ \alpha_r & \gamma_r \end{bmatrix}; \alpha_r = M_r^{-1}(\mathrm{q}_r)\bar{M}_r(\mathrm{q}_r)F_{43};$$
$$\gamma_r = M_r^{-1}(\mathrm{q}_r)\left(\bar{M}_r(\mathrm{q}_r)\right)F_{44} - \Delta C_r\left(q_r, \dot{q}_r\right)B_r^{-1}$$

For robust stability and position coordination, the controller parameters should satisfy the conditions in theorems 1 and 2. Since, regarding property 1 and Eq. (21), the variable parameters α_i and γ_i in $\tilde{F}_l$ and $\tilde{F}$ are bounded as $\underline{\alpha}_i < \alpha_i < \bar{\alpha}_i, \underline{\gamma}_i < \gamma_i < \bar{\gamma}_i$; the set of LMIs (Eqs. (14) and (15)) should be satisfied in the corners, $\underline{\alpha}_i, \bar{\alpha}_i, \underline{\gamma}_i \text{and} \bar{\gamma}_i$.

Proposition1 is presented to verify the transparency of the proposed scheme.

Proposition 1. Consider the teleoperator described with Eqs. (3) and (4) controlled by Eqs. (9) and (10). In the steady state (i.e., $\dot{q}_i = \ddot{q}_i = 0$), the human operator feels what the remote manipulator is touching (i.e. $\tau_h = \tau_e$) if the controller parameters F_{21}, F_{43} and K_i satisfy in

$$M_l F_{21} K_l = M_r F_{43} K_r \tag{23}$$

Proof. This proposition is easily established if we rewrite the teleoperator dynamic (Eqs. (3) and (4)) with controller (Eqs. (12) and (13)) in steady state as

$$\tau_h = \frac{1}{2} M_l F_{21} K_l\left(x_{1l} - x_{1r}\left(t - T_r\right)\right) = \tau_e \tag{24}$$

4- SIMULATION RESULTS

In this part, the teleoperator controlled by the proposed controller are simulated in Matlab®. The local and remote robots are 2-DOF manipulators, as shown in Fig. 2. The dynamics of the system is as Eq. (1) with the following inertia matrix adopted from reference [7]

$$M_i\left(q_i\right) = \begin{bmatrix} \alpha_i + 2\beta_i \cos\left(q_{2_i}\right) & \delta_i + \beta_i \cos\left(q_{2_i}\right) \\ \delta_i + \beta_i \cos\left(q_{2_i}\right) & \delta_i \end{bmatrix}$$

where $q_{k_i}, k \in \{1,2\}$ is the angular position of link, $\alpha_i := l_{2_i}^2 m_{2_i} + l_{1_i}^2\left(m_{1_i} + m_{2_i}\right)$, $\beta_i := l_{1_i} l_{2_i} m_{2_i}$ and $\delta_i := l_{2_i}^2 m_{2_i}$. The lengths of links l_{1_i} and l_{2_i} are 0.38 m and the masses of links are $m_{1_i} = 3.9473\,\mathrm{kg}, m_{2_i} = 0.6232$ kg. The Coriolis and centrifugal forces are shown by the matrix $C_i\left(q_i, \dot{q}_i\right)$ as

$$C_i\left(q_i, \dot{q}_i\right)\dot{q}_i = \begin{bmatrix} -\beta_i \sin\left(q_{2_i}\right)\dot{q}_{2_i} & -\beta_i \sin\left(q_{2_i}\right)\dot{q}_{1_i} \\ \beta_i \sin\left(q_{2_i}\right) & 0 \end{bmatrix}$$

where $\dot{q}_{k_i}$, $k \in \{1,2\}$ is the angular velocity of link. The gravity is given by

$$g_i\left(q_i\right) = \begin{bmatrix} \frac{1}{l_{2_i}} g\ \delta_i \cos\left(q_{1_i} + q_{2_i}\right) + \frac{1}{l_{1_i}}\left(\alpha_i - \delta_i\right)\cos\left(q_{1_i}\right) \\ \frac{1}{l_{2_i}} g\ \delta_i \cos\left(q_{1_i} + q_{2_i}\right) \end{bmatrix}$$

with g=9.8.

The time-delays in forward and backward paths vary as shown in Fig. 3. So, the lower and upper bounds of delays are $h_l = 0.8, h_r = 0.70$ and the upper bounds of their derivatives are $\mu_l = 0.72, \mu_r = 0.68$.

The interaction forces between human and local manipulator and between environment and remote manipulator are considered to be non-passive as follows

$$\tau_h = \tau_{h0} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} q_l + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \dot{q}_l$$
$$\tau_e = \tau_{e0} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} q_r - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \dot{q}_r \tag{25}$$

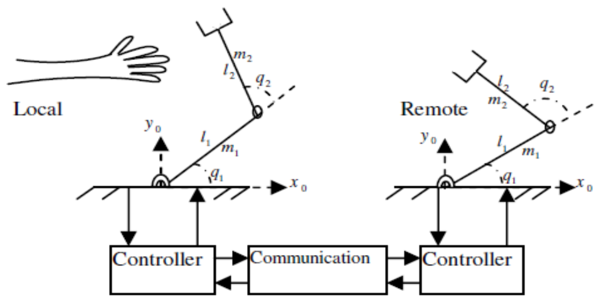


Fig. 2. Schematic of simulated 2-DOF teleoperation system [7]

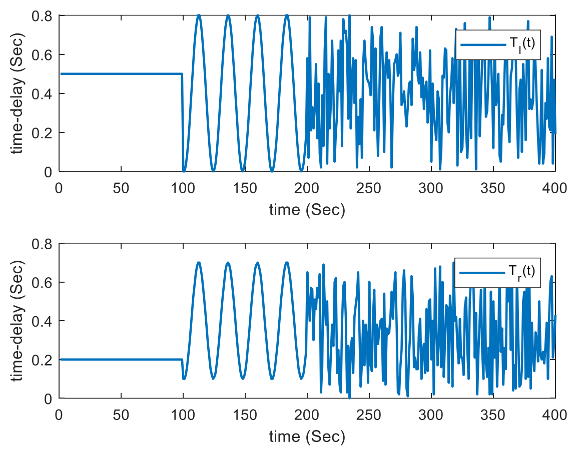


Fig. 3. The variable time-delays in forward and backward paths.

where their constant part, τ_{h0} and τ_{e0} are as depicted in Fig. 4.

Using the LMI toolbox of MATLAB[*], the parameters of IDA controllers are obtained from Theorem 1 as the following:

$$B_l = B_r = 8\times \underline{I}, K_l = K_r = 7\times \underline{I}, \underline{I} \in R^{2\times 2}; F_{21} = F_{43} = \begin{bmatrix} -7 & 3 \\ 3 & -43 \end{bmatrix}, F_{22} = F_{44} = \begin{bmatrix} -1 & 0.3 \\ 0.3 & -10 \end{bmatrix}.$$

The position coordination of teleoperator and the velocity of manipulations are depicted in Figs. 5 and 6, respectively. The initial conditions are chosen as $x_{1_l} = x_{2_l} = x_{2_r} = [0\ 0]^T, x_{1_r} = [0.1\ 0.1]^T$ rad.

The proposed IDA-PBC controller improves the performance of coordination compared to the methods developed in references [20,22]. It is worth noting that the controller proposed in reference [20] is as

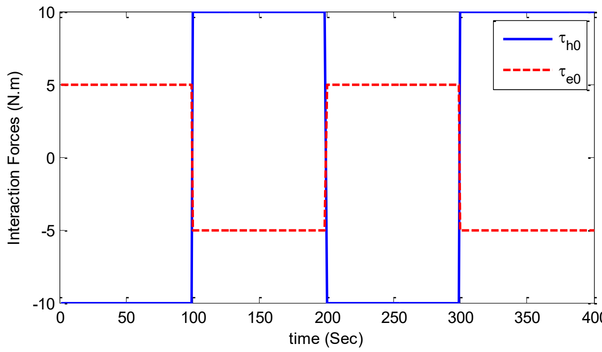


Fig. 4. The profile of the constant part of interaction forces

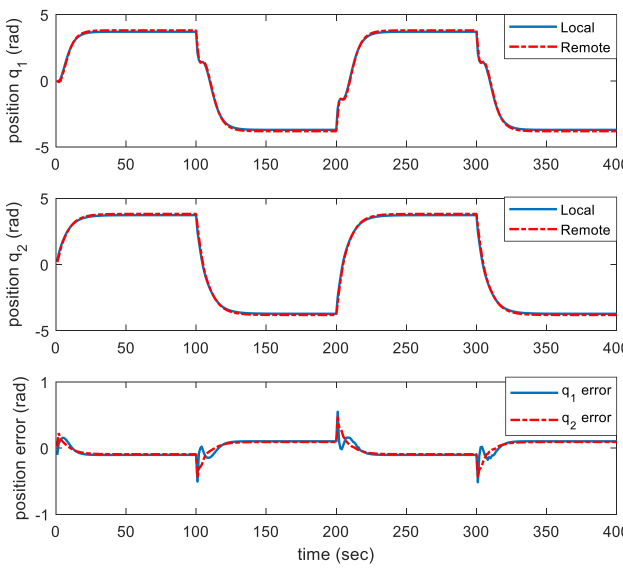


Fig. 5. Positions in the joint space for the teleoperator system controlled by the proposed method

$$\begin{aligned} \tau_l^* &= g_l(q_l) - k_d(\dot{q}_l - \dot{q}_r(t-T_r(t))) - \\ & k_p(q_l - q_r(t-T_r(t))) - \alpha_l \dot{q}_l \\ \tau_r^* &= g_r(q_r) + k_d(\dot{q}_l(t-T_l(t)) - \dot{q}_r) + \\ & k_p(q_l(t-T_l(t)) - q_r) - \alpha_r \dot{q}_r \end{aligned} \tag{26}$$

where $k_p = 1, k_d = 1.5, \alpha_l = \alpha_r = 0.2$. Also, the controller suggested in reference [22] is as

$$\begin{aligned} \tau_l^* &= g_l(q_l) + C_l(q_l, \dot{q}_l)\dot{q}_l - \\ & k_{p_l}(q_l - q_r(t-T_r(t))) - \alpha_l \dot{q}_l \\ \tau_r^* &= g_r(q_r) + C_r(q_r, \dot{q}_r)\dot{q}_r + \\ & k_{p_l}(q_l(t-T_l(t)) - q_r) - \alpha_r \dot{q}_r \end{aligned} \tag{27}$$

where $k_{p_l} = 3, \alpha_l = 0.5, \alpha_r = 0.5$. The simulation results of system with proposed controller and methods in references [20, 22] in joint space and workspace are shown in Fig. 7. Moreover, mean square errors and maximum errors are reported in Table 1, where $e_k = q_{k_l} - q_{k_r}$ for $k = 1,2$ represents position error of links in joint space. As seen, the Mean Square of position Errors (MSEs) and maximum errors obtained by our method are considerably lower than the rival ones.

To illustrate the force tracking in the system (i.e. transparency) the profiles of the force applied by the environment to the remote robot and the force felt by the operator via the local robot are shown in Fig. 8. In this part of the simulation, the external interaction force applied by the operator to the local robot is zero. As seen, the proposed control strategy can provide transparency as well as stability and tracking in the telemanipulation system interacting non-passive human and environment.

To evaluate the performance of system in the presence of uncertainties in the manipulators models, it is assumed that the knowledge about the masses of the links has 10% uncertainty. i.e., $m_{1_l} \in [3.54, 4.33]$kg and

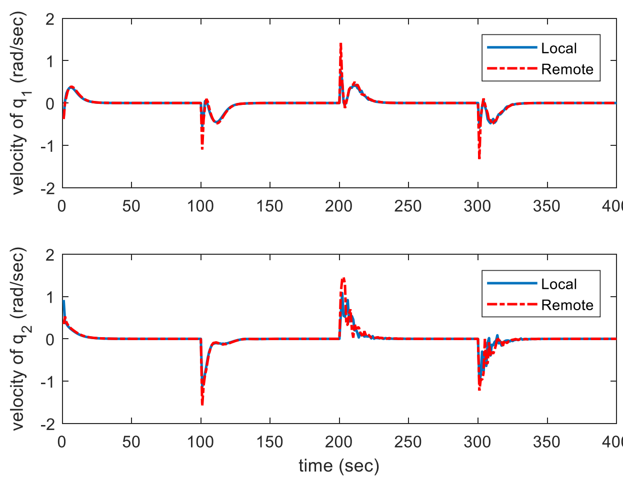
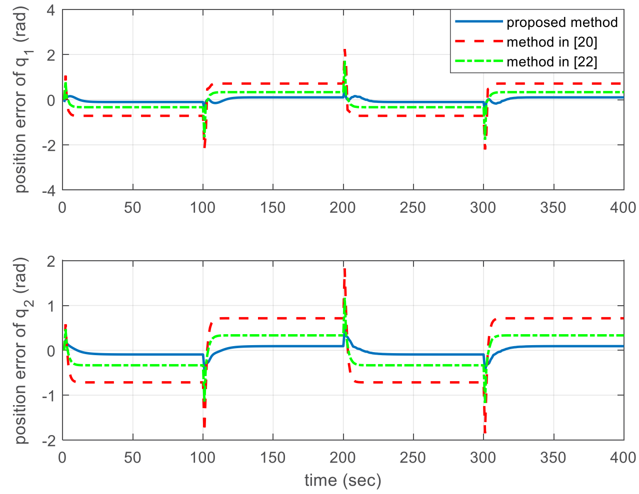
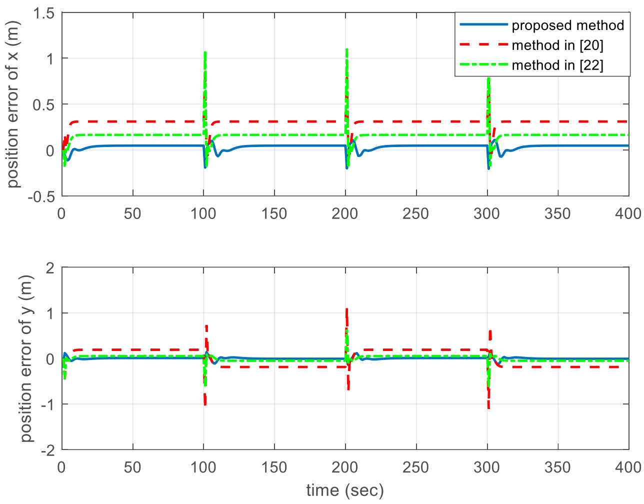


Fig. 6. Velocities in the joint space for the teleoperator system controlled by the proposed method



(a)



(b)

Fig. 7. Position errors in (a) joint space (b) workspace for the teleoperator system controlled by proposed controller and rival ones

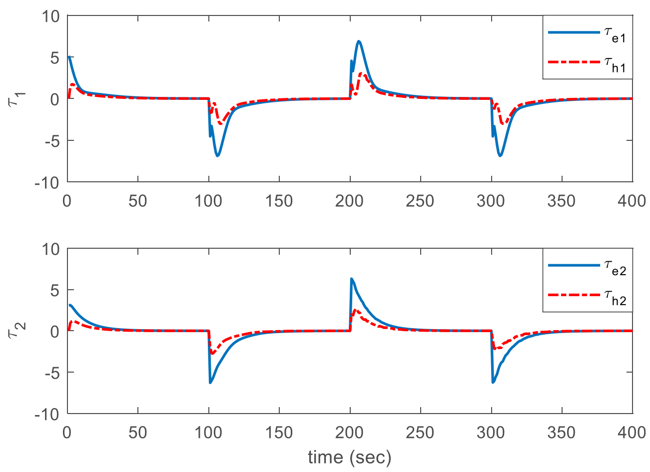


Fig. 8. Interaction forces between environment and remote robot and forces sensed by the human operator.

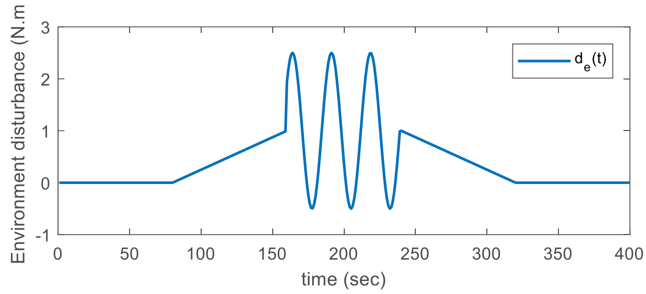


Fig. 9. Disturbance forces from environment applied on remote robot.

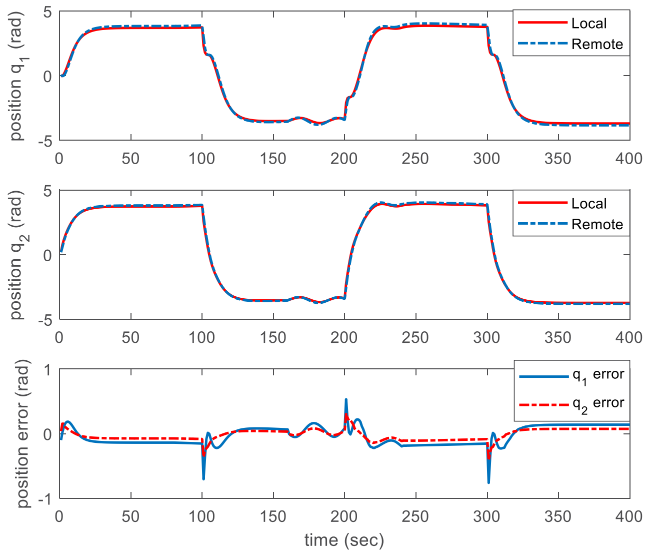


Fig. 10. Positions in the joint space for the teleoperator in the presence of model uncertainty and unknown disturbance

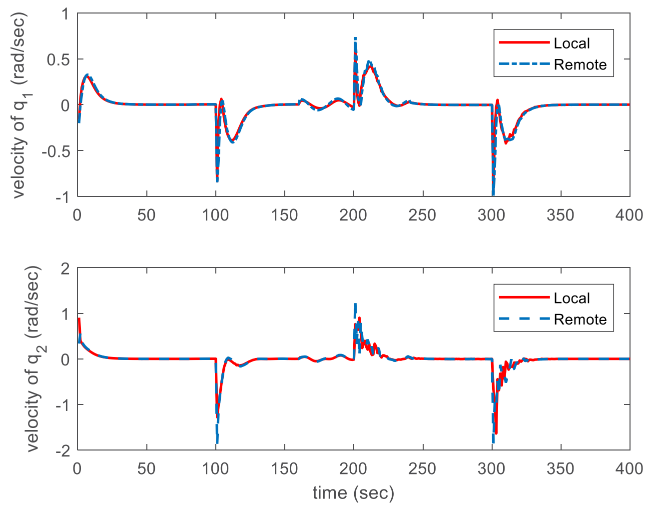


Fig. 11. Velocities in the joint space for the teleoperator in the presence of model uncertainty and unknown disturbance

Table 1. Position errors in joint space

	MSE (e_1) [rad]	MSE (e_2) [rad]	Max (e_1) [rad]	Max (e_2) [rad]
Proposed method	0.0121	0.0111	0.605	0.493
Method in reference [20]	0.547	0.516	2.221	1.914
Method in reference [22]	0.129	0.116	1.810	1.28

$m_{2_i} \in [0.55, 0.68]$kg. In this case, the uncertainty bound of system matrices by assumption that $|\dot{q}_i| \le 15$ rad/sec are $\delta_M = 0.096, \delta_C = 0.324, \delta_g = 0.98$. From property1, it is clear that $0.047I < M_i(\mathrm{q}_i) < 1.06I$ and $0.052I < \bar{M}_i(\mathrm{q}_i) < 0.96I, I \in \mathbb{R}^{2\times 2}$ Regarding Remark 2, the parameters of IDA controllers for closed-loop system (Eq. (22)) are obtained from Theorem 1 as below:

$$B_l = B_r = 8 \times \underline{I}, K_l = K_r = 9 \times \underline{I}, \underline{I} \in R^{2\times 2}; F_{21} = F_{43} = \begin{bmatrix} -7 & 3 \\ 3 & -60 \end{bmatrix}, F_{22} = F_{44} = \begin{bmatrix} -4 & 0.3 \\ 0.3 & -13 \end{bmatrix}.$$

The performance of system in the presence of uncertainty and some unknown disturbance (depicted in Fig. 9) for the remote robot, is shown in Figs. 10 and 11. As seen, the effects of uncertainties are compensated by the controller and system has appropriate behavior. Compared to the deterministic case, the unknown disturbance on the remote robot has decreased the performance of system but stability of system is preserved.

5- CONCLUSIONS

In this paper, the notion of IDA-PBC has been employed for nonlinear bilateral teleoperation systems with asymmetric variable time-delay in the communication medium and non-passive operator and environment. First, using the Lyapunov-Krasovskii theorem, the delay-dependent conditions have been extracted which is used to tune the parameters of IDA-PBC in order to achieve stable position and force tracking in system. Employing the Lyapunov redesign scheme, another control term has been added to assure the stability of system in spite of non-passive interaction forces. Unlike literature dynamical model of these forces in not needed in the design procedure. Comparative simulations show that by the proposed approach, the position tracking of system is improved compared to P+d based controllers. Considering more imperfections in the communication channel defines future research line.

Appendix A:

Proof of Theorem 1. The LK functional candidate is as below:

$$V(t, x_t) := 2H_d(x, \tilde{x}_l, \tilde{x}_r) + V_1(t, x_t) + V_2(t, x_t) + V_3(t, x_t) \tag{A1}$$

where $x_t := x(t+\theta)$, for $-2\max\{h_l, h_r\} \le \theta \le 0$, and

$$V_1 := \nabla_x^T H_d(x, \tilde{x}_l, \tilde{x}_r) P \nabla_x H_d(x, \tilde{x}_l, \tilde{x}_r)$$

$$V_2 := \int_{t-T_l(t)}^{t} \nabla_x^T H_d(x(s), \tilde{x}_l(s), \tilde{x}_r(s)) Q_1 \nabla_x H_d(x(s), \tilde{x}_l(s), \tilde{x}_r(s)) ds + \int_{t-T_r(t)}^{t} \nabla_x^T H_d(x(s), \tilde{x}_l(s), \tilde{x}_r(s)) Q_2 \nabla_x H_d(x(s), \tilde{x}_l(s), \tilde{x}_r(s))\, ds$$

$$V_3 := h_l \int_{t-h_l}^{t} \int_{s}^{t} \nabla_x^T H_d(x(\tau), \tilde{x}_l(\tau), \tilde{x}_r(\tau)) R_1 \nabla_x H_d(x(\tau), \tilde{x}_l(\tau), \tilde{x}_r(\tau)) d\tau ds + h_r \int_{t-h_r}^{t} \int_{s}^{t} \nabla_x^T H_d(x(\tau), \tilde{x}_l(\tau), \tilde{x}_r(\tau)) R_2 \nabla_x H_d(x(\tau), \tilde{x}_l(\tau), \tilde{x}_r(\tau)) d\tau ds$$

The derivative of $2H_d(x, \tilde{x}_l, \tilde{x}_r)$ along the system (Eq. (12)) is calculated as

$$\begin{aligned} 2\dot{H}_d(x, \tilde{x}_l, \tilde{x}_r) &= 2\begin{pmatrix} \nabla_x{}^T H_d(x, \tilde{x}_l, \tilde{x}_r)\dot{x} + \\ \nabla_{\tilde{x}_l}{}^T H_d(x, \tilde{x}_l, \tilde{x}_r)\dot{\tilde{x}}_l + \\ \nabla_{\tilde{x}_r}{}^T H_d(x, \tilde{x}_l, \tilde{x}_r)\dot{\tilde{x}}_r \end{pmatrix} \\ &= \nabla_x^T H_d(x, \tilde{x}_l, \tilde{x}_r)(F_d + F_d^T)\nabla_x H_d(x, \tilde{x}_l, \tilde{x}_r) \\ &\quad + 2\underbrace{\nabla_{\tilde{x}_l}{}^T H_d(x, \tilde{x}_l, \tilde{x}_r)}_{a^T} F_d \underbrace{\nabla_{\tilde{x}_l} H_d(\tilde{x}_l, \tilde{\tilde{x}}_l, \tilde{\tilde{x}}_{rl})}_{b} + \\ &\quad 2\underbrace{\nabla_{\tilde{x}_r}{}^T H_d(x, \tilde{x}_l, \tilde{x}_r)}_{a^T} F_d \underbrace{\nabla_{\tilde{x}_r} H_d(\tilde{x}_r, \tilde{\tilde{x}}_{rl}, \tilde{\tilde{x}}_r)}_{b} \end{aligned} \tag{A2}$$

where $\tilde{\tilde{x}}_l := x(t - 2T_l),\ \tilde{\tilde{x}}_r := x(t - 2T_r), \tilde{\tilde{x}}_{rl} := x(t - T_l - T_r)$.

Using the inequality $2a^T b \le a^T a + b^T b$ leads to

$$\begin{aligned}&2\dot{H}_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right) \le \nabla_x^T H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)\\&\left(F_d + F_d^T\right)\nabla_x H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)\\&+\nabla_{\tilde{x}_l}^T H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)\nabla_{\tilde{x}_l} H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)+\\&\nabla_{\tilde{x}_l}^T H_d\left(\tilde{x}_l,\,\tilde{\tilde{x}}_l,\,\tilde{\tilde{x}}_{rl}\right)F_d^T F_d \nabla_{\tilde{x}_l} H_d\left(\tilde{x}_l,\,\tilde{\tilde{x}}_l,\,\tilde{\tilde{x}}_{rl}\right)\\&+\nabla_{\tilde{x}_r}^T H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)\nabla_{\tilde{x}_r}^T H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)+\\&\nabla_{\tilde{x}_r}^T H_d\left(\tilde{x}_r,\,\tilde{\tilde{x}}_{rl},\,\tilde{\tilde{x}}_r\right)F_d^T F_d \nabla_{\tilde{x}_r} H_d\left(\tilde{x}_r,\,\tilde{\tilde{x}}_{rl},\,\tilde{\tilde{x}}_r\right)\end{aligned} \tag{A3}$$

According to Eqs. (13) and (14), we have

$$\begin{aligned}&\nabla_{\tilde{x}_l}^T H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)\nabla_{\tilde{x}_l} H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)=\\&\begin{bmatrix}x\\\tilde{x}_l\\\tilde{x}_r\end{bmatrix}^T F_2^T F_2 \begin{bmatrix}x\\\tilde{x}_l\\\tilde{x}_r\end{bmatrix} \le \begin{bmatrix}x\\\tilde{x}_l\\\tilde{x}_r\end{bmatrix}^T F_1^T Z_1 F_1 \begin{bmatrix}x\\\tilde{x}_l\\\tilde{x}_r\end{bmatrix}\\&\le \nabla_x^T H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)Z_1\nabla_x H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)\end{aligned} \tag{A4}$$

In similar way, we obtain

$$\begin{aligned}&\nabla_x^T H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)\left(F_d + F_d^T\right)\nabla_x H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)\\&\le \nabla_x^T H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)L\nabla_x H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right);\\&\nabla_{\tilde{x}_r}^T H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)\nabla_{\tilde{x}_r} H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)\\&\le \nabla_x^T H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)Z_2\nabla_x H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right);\\&\nabla_{\tilde{x}_l}^T H_d\left(\tilde{x}_l,\,\tilde{\tilde{x}}_l,\,\tilde{\tilde{x}}_{rl}\right)F_d^T F_d \nabla_{\tilde{x}_l} H_d\left(\tilde{x}_l,\,\tilde{\tilde{x}}_l,\,\tilde{\tilde{x}}_{rl}\right)\\&\le \nabla_{\tilde{x}_l}^T H_d\left(\tilde{x}_l,\,\tilde{\tilde{x}}_l,\,\tilde{\tilde{x}}_{rl}\right)S\nabla_{\tilde{x}_l} H_d\left(\tilde{x}_l,\,\tilde{\tilde{x}}_l,\,\tilde{\tilde{x}}_{rl}\right);\\&\nabla_{\tilde{x}_r}^T H_d\left(\tilde{x}_r,\,\tilde{\tilde{x}}_{rl},\,\tilde{\tilde{x}}_r\right)F_d^T F_d \nabla_{\tilde{x}_r} H_d\left(\tilde{x}_r,\,\tilde{\tilde{x}}_{rl},\,\tilde{\tilde{x}}_r\right)\\&\le \nabla_{\tilde{x}_r}^T H_d\left(\tilde{x}_r,\,\tilde{\tilde{x}}_{rl},\,\tilde{\tilde{x}}_r\right)S\nabla_{\tilde{x}_r} H_d\left(\tilde{x}_r,\,\tilde{\tilde{x}}_{rl},\,\tilde{\tilde{x}}_r\right)\end{aligned} \tag{A5}$$

Combining the above inequalities results in the upper bound for $2H_d\left(x,\tilde{x}_l,\tilde{x}_r\right)$ as below

$$\begin{aligned}&2\dot{H}_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right) \le \nabla_x^T H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)\\&\left(L + Z_1 + Z_2\right)\nabla_x H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)\\&+\nabla_{\tilde{x}_l}^T H_d\left(\tilde{x}_l,\,\tilde{\tilde{x}}_l,\,\tilde{\tilde{x}}_{rl}\right)S\nabla_{\tilde{x}_l} H_d\left(\tilde{x}_l,\,\tilde{\tilde{x}}_l,\,\tilde{\tilde{x}}_{rl}\right)+\\&\nabla_{\tilde{x}_r}^T H_d\left(\tilde{x}_r,\,\tilde{\tilde{x}}_{rl},\,\tilde{\tilde{x}}_r\right)S\nabla_{\tilde{x}_r} H_d\left(\tilde{x}_r,\,\tilde{\tilde{x}}_{rl},\,\tilde{\tilde{x}}_r\right)\end{aligned} \tag{A6}$$

On the other hand, the time derivative of V_1, V_2 along the system Eq. (12) is computed as

$$\begin{aligned}&\dot{V}_1 = 2\nabla_x^T H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)\\&P\left(\begin{aligned}&\overbrace{\nabla_{xx} H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)}^{A}\dot{x}+\\&\overbrace{\nabla_{x\tilde{x}_l} H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)}^{B}\dot{\tilde{x}}_l+\\&\overbrace{\nabla_{x\tilde{x}_r} H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)}^{C}\dot{\tilde{x}}_r\end{aligned}\right)\\&= 2\nabla_x^T H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)PAF_d\nabla_x H_d\\&\left(x,\tilde{x}_l,\,\tilde{x}_r\right)+2\nabla_x^T H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)\\&\mathrm{P}BF_d\nabla_{\tilde{x}_l} H_d\left(\tilde{x}_l,\,\tilde{\tilde{x}}_l,\,\tilde{\tilde{x}}_{rl}\right)\\&+2\nabla_x^T H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)\\&PCF_d\nabla_{\tilde{x}_r} H_d\left(\tilde{x}_r,\,\tilde{\tilde{x}}_{rl},\,\tilde{\tilde{x}}_r\right);\end{aligned} \tag{A7}$$

$$\begin{aligned}&\dot{V}_2 = \nabla_x^T H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)\\&\left(Q_1 + Q_2\right)\,\nabla_x H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)\\&-\left(1-\dot{T}_l\right)\nabla_{\tilde{x}_l}^T H_d\left(\tilde{x}_l,\,\tilde{\tilde{x}}_l,\,\tilde{\tilde{x}}_{rl}\right)\\&Q_1\nabla_{\tilde{x}_l} H_d\left(\tilde{x}_l,\,\tilde{\tilde{x}}_l,\,\tilde{\tilde{x}}_{rl}\right)\\&-\left(1-\dot{T}_r\right)\nabla_{\tilde{x}_r}^T H_d\left(\tilde{x}_r,\,\tilde{\tilde{x}}_{rl},\,\tilde{\tilde{x}}_r\right)\\&Q_2\nabla_{\tilde{x}_r} H_d\left(\tilde{x}_r,\,\tilde{\tilde{x}}_{rl},\,\tilde{\tilde{x}}_r\right)\end{aligned} \tag{A8}$$

Regarding delay characteristics, the upper bound of $\dot{V}_2$ is

$$\begin{aligned}&\dot{V}_2 \le \nabla_x^T H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)\\&\left(Q_1 + Q_2\right)\,\nabla_x H_d\left(x,\tilde{x}_l,\,\tilde{x}_r\right)\\&-\left(1-\mu_l\right)\nabla_{\tilde{x}_l}^T H_d\left(\tilde{x}_l,\,\tilde{\tilde{x}}_l,\,\tilde{\tilde{x}}_{rl}\right)\\&Q_1\nabla_{\tilde{x}_l} H_d\left(\tilde{x}_l,\,\tilde{\tilde{x}}_l,\,\tilde{\tilde{x}}_{rl}\right)\\&-\left(1-\mu_r\right)\nabla_{\tilde{x}_r}^T H_d\left(\tilde{x}_r,\,\tilde{\tilde{x}}_{rl},\,\tilde{\tilde{x}}_r\right)\\&Q_2\nabla_{\tilde{x}_r} H_d\left(\tilde{x}_r,\,\tilde{\tilde{x}}_{rl},\,\tilde{\tilde{x}}_r\right)\end{aligned} \tag{A9}$$

$\dot{V}_3$ is calculated as

$$\dot{V}_3 = h_l \int_{t-h_l}^{t} \left\{ \begin{matrix} \nabla_x^T H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) \\ R_1 \nabla_x H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) - \\ \nabla_x^T H_d\left(x(s),\tilde{x}_l(\mathrm{s}),\tilde{x}_r(\mathrm{s})\right) \\ R_1 \nabla_x H_d\left(x(s),\tilde{x}_l(\mathrm{s}),\tilde{x}_r(s)\right) \end{matrix} \right\} ds$$
$$+h_r \int_{t-h_r}^{t} \left\{ \begin{matrix} \nabla_x^T H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) \\ R_2 \nabla_x H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) - \\ \nabla_x^T H_d\left(x(s),\tilde{x}_l(\mathrm{s}),\tilde{x}_r(\mathrm{s})\right) \\ R_2 \nabla_x H_d\left(x(s),\tilde{x}_l(\mathrm{s}),\tilde{x}_r(s)\right) \mathrm{ds} \end{matrix} \right\} \mathrm{ds} \qquad \text{(A10)}$$

Some manipulations yields to

$$\begin{aligned} \dot{V}_3 = {} & h_l^2 \nabla_x^T H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) R_1 \nabla_x H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) \\ & -h_l \int_{t-h_l}^{t} \nabla_x^T H_d\left(x(s),\tilde{x}_l(\mathrm{s}),\tilde{x}_r(\mathrm{s})\right) R_1 \nabla_x H_d\left(x(s),\tilde{x}_l(\mathrm{s}),\tilde{x}_r(s)\right) \mathrm{ds} \\ & +h_r^2 \nabla_x^T H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) R_2 \nabla_x H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) \\ & -h_r \int_{t-h_r}^{t} \nabla_x^T H_d\left(x(s),\tilde{x}_l(\mathrm{s}),\tilde{x}_r(\mathrm{s})\right) R_2 \nabla_x H_d\left(x(s),\tilde{x}_l(\mathrm{s}),\tilde{x}_r(s)\right) \mathrm{ds} \end{aligned} \qquad \text{(A11)}$$

So, the upper bound of $\dot{V}_3$ is obtained as below

$$\begin{aligned} \dot{V}_3 \le {} & h_l^2 \nabla_x^T H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) R_1 \nabla_x H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) \\ & + h_r^2 \nabla_x^T H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) R_2 \nabla_x H_d\left(x,\tilde{x}_l,\tilde{x}_r\right); \end{aligned} \qquad \text{(A12)}$$

Finally, regarding Eqs. (A6), (A7), (A9) and (A12), the upper bound for $\dot{V}$ is obtained as

$$\begin{aligned} \dot{V} \le {} & \nabla_x^T H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) \left(\overbrace{\begin{matrix} L + Z_1 + Z_2 + PAF_d + \left(PAF_d\right)^{\mathrm{T}} \\ +Q_1 + Q_2 + h_l^2 R_1 + h_r^2 R_2 \end{matrix}}^{\Gamma_{11}} \right) \nabla_x H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) \\ & +2\nabla_x^T H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) \left(\overbrace{PBF_d}^{\Gamma_{12}} \right) \nabla_{\tilde{x}_l} H_d\left(\tilde{x}_l,\tilde{\tilde{x}}_l,\tilde{\tilde{x}}_{rl}\right) + 2\nabla_x^T H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) \\ & \left(\overbrace{PCF_d}^{\Gamma_{13}} \right) \nabla_{\tilde{x}_r} H_d\left(\tilde{x}_r,\tilde{\tilde{x}}_{rl},\tilde{\tilde{x}}_r\right) \\ & +\nabla_{\tilde{x}_l}^T H_d\left(\tilde{x}_l,\tilde{\tilde{x}}_l,\tilde{\tilde{x}}_{rl}\right) \left(\overbrace{-\left(1-\mu_l\right)Q_1 + S}^{\Gamma_{22}} \right) \nabla_{\tilde{x}_l} H_d\left(\tilde{x}_l,\tilde{\tilde{x}}_l,\tilde{\tilde{x}}_{rl}\right) \\ & +\nabla_{\tilde{x}_r}^T H_d\left(\tilde{x}_r,\tilde{\tilde{x}}_{rl},\tilde{\tilde{x}}_r\right) \overbrace{\left(-\left(1-\mu_r\right)Q_2 + S\right)}^{\Gamma_{33}} \nabla_{\tilde{x}_r} H_d\left(\tilde{x}_r,\tilde{\tilde{x}}_{rl},\tilde{\tilde{x}}_r\right) \end{aligned} \qquad \text{(A13)}$$

That can be expressed in compact form as

$$-D\left(x,\tilde{x}_l,\tilde{x}_r\right) := \begin{bmatrix} \nabla_x H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) \\ \nabla_{\tilde{x}_l} H_d\left(\tilde{x}_l,\tilde{\tilde{x}}_l,\tilde{\tilde{x}}_{rl}\right) \\ \nabla_{\tilde{x}_r} H_d\left(\tilde{x}_r,\tilde{\tilde{x}}_{rl},\tilde{\tilde{x}}_r\right) \end{bmatrix}^T \underbrace{\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} \\ * & \Gamma_{22} & \underline{0} \\ * & * & \Gamma_{33} \end{bmatrix}}_{\Gamma} \begin{bmatrix} \nabla_x H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) \\ \nabla_{\tilde{x}_l} H_d\left(\tilde{x}_l,\tilde{\tilde{x}}_l,\tilde{\tilde{x}}_{rl}\right) \\ \nabla_{\tilde{x}_r} H_d\left(\tilde{x}_r,\tilde{\tilde{x}}_{rl},\tilde{\tilde{x}}_r\right) \end{bmatrix} \qquad \text{(A14)}$$

By LK Theorem, the system Eq. (12) is locally asymptotically stable if the upper bound of $\dot{V}$ is negative. By attention to Eq. (A14) the upper bound of $\dot{V}$ is negative if the matrix $\Gamma < 0$, thus the proof is completed.

Proof of Theorem 2. Consider again the LK functional (E. (A1)). Calculating the derivative of $V(t,x_t)$ along the trajectories of the system Eq. (17), yields to

$$\begin{aligned} \dot{V} \le {} & -D\left(x,\tilde{x}_l,\tilde{x}_r\right) - 2\nabla_x^T H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) g(x)(w+v) \\ & -2\nabla_{\tilde{x}_l}^T H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) g\left(\tilde{x}_l\right)\left(\tilde{w}_l + \tilde{v}_l\right) \\ & -2\nabla_{\tilde{x}_r}^T H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) g\left(\tilde{x}_r\right)\left(\tilde{w}_r + \tilde{v}_r\right) \\ & -2\nabla_x^T H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) PAg(x)(w+v) \\ & -2\nabla_x^T H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) PBg\left(\tilde{x}_l\right)\left(\tilde{w}_l + \tilde{v}_l\right) \\ & -2\nabla_x^T H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) PCg\left(\tilde{x}_r\right)\left(\tilde{w}_r + \tilde{v}_r\right) \end{aligned} \qquad \text{(A15)}$$

which is rewritten as

$$\begin{aligned}
\dot{V} \le & -D\left(x,\tilde{x}_l,\tilde{x}_r\right) \\
& \underbrace{-2\nabla_x^T H_d\left(x,\tilde{x}_l,\tilde{x}_r\right)\left(I+PA\right)g\left(x\right)}_{\omega_1^T}\omega\left(w+v\right) \\
& \underbrace{-2\begin{pmatrix}\nabla_{\tilde{x}_l}^T H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) \\ +\nabla_x^T H_d\left(x,\tilde{x}_l,\tilde{x}_r\right)PB\end{pmatrix} g\left(\tilde{x}_l\right)}_{\omega_2^T}\left(\tilde{w}_l+\tilde{v}_l\right) \\
& \underbrace{-2\begin{pmatrix}\nabla_{\tilde{x}_r}^T H_d\left(x,\tilde{x}_l,\tilde{x}_r\right) \\ +\nabla_x^T H_d\left(x,\tilde{x}_l,\tilde{x}_r\right)PC\end{pmatrix} g\left(\tilde{x}_r\right)}_{\omega_3^T}\left(\tilde{w}_r+\tilde{v}_r\right)
\end{aligned} \quad \text{(A16)}$$

where $\tilde{w}_l := w\left(t-T_l\right)$, $\tilde{w}_r := w\left(t-T_r\right)$, $\tilde{v}_l := v\left(t-T_l\right)$, $\tilde{v}_r := v\left(t-T_r\right)$. Defining $\omega^T = \left[\omega_1^T, \omega_2^T, \omega_3^T\right], W_w = \left[w^T, \tilde{w}_l^T, \tilde{w}_r^T\right]^T$ and $V_v = \left[v^T, \tilde{v}_l^T, \tilde{v}_r^T\right]^T$, Eq. (A16) is rewritten as $\dot{V} \le -D\left(x,\tilde{x}_l,\tilde{x}_r\right) + \omega^T W_w + \omega^T V_v$, Since $\left\|W_w\right\| \le \rho$, we have

$$\begin{aligned}
& \omega^T W_w + \omega^T V_v \le \left\|\omega\right\|\left\|W_w\right\| \\
& +\omega^T V_v \le \rho\left\|\omega\right\| + \omega^T V_v
\end{aligned} \quad \text{(A17)}$$

If the auxiliary controller v is considered as

$$v := v\left(x,\tilde{x}_l,\tilde{x}_r\right) = -\eta\frac{\omega_1}{\left\|\omega\right\|} \quad \text{(A18)}$$

Then, $V_v = -\eta\frac{\omega}{\omega}$; so, the following holds

$$\omega^T W_w + \omega^T V_v \le \rho\left\|\omega\right\| - \eta\left\|\omega\right\| \quad \text{(A19)}$$

By choosing $\eta \ge \rho$, we have $\omega^T W_w + \omega^T V_v \le 0$, which yields to $\dot{V} \le 0$. The stability of the states of system Eq. (17) is proved.

NOMENCLATURE

$\mathbb{R}$	The set of real numbers
$\mathbb{R}^n$	n-dimensional real vector space
$\mathbb{R}^{n\times m}$	$n \times m$ -dimensional real matrix space
$g^{\perp}$	The full-rank left annihilator matrix of g i.e., $g^{\perp}g = 0$
$\nabla_x H$	The gradient of H with respect to x. i.e., $\frac{\partial H}{\partial x}$
$\underline{0} \in \mathbb{R}^{n\times m}$	$n \times m$ -dimensional zero matrix
$\underline{I} \in \mathbb{R}^{n\times n}$	$n \times n$ -dimensional identity matrix

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