



## Modeling of an Upper-Convected-Maxwell Fluid Hammer Phenomenon in Pipe System

B. Norouzia<sup>1</sup>, A. Ahmadi<sup>1\*</sup>, M. Norouzi<sup>2</sup>, M. Lashkarbolook<sup>3</sup>

<sup>1</sup> Faculty of Civil Engineering, Shahrood University of Technology, Shahrood, Iran

<sup>2</sup> Faculty of Mechanical Engineering, Shahrood University of Technology, Shahrood, Iran

<sup>3</sup> Department of Civil Engineering, Golestan University, Gorgan, Iran

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**ABSTRACT:** In this paper, the occurrence of water hammer phenomenon is examined in a situation that instead of water, an upper-convected-Maxwell fluid flows in a pipe system. This phenomenon is called an upper-convected-Maxwell fluid hammer. This expression relates to transients of Maxwell fluid caused by the sudden alteration in the conditions of flow. Upper-convected-Maxwell fluids are a kind of non-Newtonian viscoelastic fluids. The system studied is a valve-horizontal pipe and reservoir. The equations representing the conservation of mass and momentum govern the transitional flow in the pipe system. The numerical method used is a two-step variant of the Lax-Friedrichs method. Firstly, the non-dimensional form of governing equations is defined, then, the effect of Deborah and Reynolds numbers on pressure historic is investigated. The results revealed that increasing Deborah number, indicating the elasticity of the polymer, increases the oscillation height and consequently attenuation time of the transient flow becomes longer. It was also found that in low Reynolds, in a Newtonian fluid, line packing phenomenon effect is observed only at the first time period but in upper convected Maxwell fluid the effect of this phenomenon continues to more time periods and damping time becomes longer.

### 1- Introduction

The transitional flows due to the water-hammer phenomenon are classified in the group of unsteady flows. The hydraulic of transitional flow for the first time was first studied by Newton and Lagrange in the seventeenth century. Then, studies in this field continued widely. During the first quarter of the twentieth century, most studies on fluid hammer have been in the continent of Europe. Most of the studied concepts were related to the surge issue which was published by Joukowsky. He also achieved the formula for calculating the pressure due to the instantaneous closure of valve in the reservoir-pipe-valve system. Accordingly, numerous researchers have examined the fluid hammer phenomenon using various numerical methods. A detailed review of this matter has been given by Ghidaoui et al. [1] Computational techniques for this type of fluids is built on the mass and momentum equations, which is easily solvable using the characteristics approach. They generally provide an accurate prediction of the maximum pressure rise which usually occurs during the first pressure peak. Wylie et al. [2] developed a numerical model by using a constant value of turbulent friction factor. Multiyear study of the scientists has resulted in developing numerical models of hydraulic transients with the unsteady friction. The progress of the first group of models was originated in 1968 by Zielke [3]. In his study, an equation was derived, which related the wall shear stress in transitional laminar pipe flow to the instantaneous

mean velocity and to the weighted past velocity changes. Zielke [3] concluded that frequency-dependent effects of viscosity can be included into the one dimensional model of transient flow using the Method Of Characteristics (MOC). Vardy and Hwang [4] developed a quasi two-dimensional model of transient flows using the one dimensional method of characteristics in concentric cylindrical. They showed that Zielke's [3] model provides a good first approximation for transient turbulent flows. Bergant et al. [5] combined two unsteady friction models proposed by Zielke [3] and Brunone et al. [6] into MOC water hammer analysis. They compared the numerical results achieved for pressure heads with the results of measurements of fast valve closure in a laboratory device with laminar and turbulent flow condition. Brunelli [7] presented a model which gives the two-dimensional velocity profile in the time domain for an unstationary pipe flow. He tested his model on the experimental results presented by Zielke [3] and the agreement of his simulation with the experiment was satisfactory. Shamloo et al. [8] presented a review of unsteady friction models for transient pipe flow. They considered models which instantaneous wall shear stress is the sum of the quasi-steady value plus a term in which certain weights are given to the past velocity changes such as Zielke [3], Vardy and Brown [9] and Trikha [10] in one group and other models in another group and concluded that Zielke [3] model yields better conformance with the experimental data. Zhao [11] proposed the modified Chebyshev polynomial expansions of the radial distribution of the axial velocity and radial component flux for solving the quasi-two-dimensional equations of the water-hammer problem under laminar flow

\*Corresponding author's email: a.ahmadi@shahroodut.ac.ir



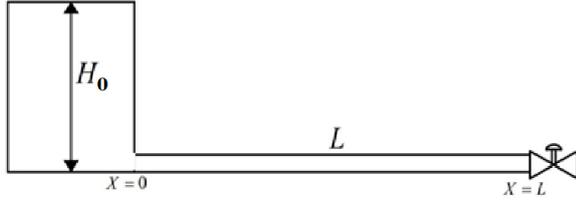


Fig. 1. Schematic of the reservoir-pipe-valve system

conditions. The results obtained from his proposed model are almost coincident with those obtained from the model of Vardy and Hwang [4] and both agree with the experimental data well. In this work, the authors consider the case in which, instead of water, a kind of non-Newtonian viscoelastic fluid called Upper-Convected-Maxwell (UCM) fluid flows in the pipe system. Unlike Newtonian fluid hammer, there is not a great amount of work on the non-Newtonian fluids. Wahba [12] studied shear-thinning and shear-thickening effects on non-Newtonian fluid hammer in elastic circular pipes. He concluded that the shear-thickening behavior results in more rapid attenuation of the fluid transient and excessive pipeline packing which would lead to a pressure rise at the valve that would significantly exceed the inviscid Joukowski pressure rise. Besides, An upper-convected-Maxwell fluid is a type of viscoelastic fluids. Viscoelastic fluids are a common form of Non-Newtonian fluids. They can exhibit a response that resembles that of an elastic solid under some conditions, or the response of a viscous liquid under other situations. In these fluids, contrary to the Newtonian ones, a linear relation between the stress tensor and the rate of deformation tensor do not hold. So, they need more complicated constitutive relationships to close the system of equations that has to be solved. Mora and Manna [13] presented an analytical and numerical study of the linear Saffman–Taylor instability for a Maxwell viscoelastic fluid. Darwish et al. [14] and later Missirlis et al. [15] used a finite-volume technique to simulate the flow of a viscoelastic liquid through a 1:4 plane sudden expansion using the upper convected Maxwell model. Poole et al. [16] reported the results of a systematic numerical investigation, using a finite volume technique, of the creeping flow ( $Re=0.01$ ) of three model viscoelastic fluids, the UCM, Oldroyd-B and the Phan-Thien-Tanner (PTT) models. In general, and up to the present author’s best knowledge, it seems to be a lack of studies investigating fluid transients with upper convected Maxwell fluids in pipes. These fluids are used as melted polymers in chemical and food industries to produce specific products. Considering that the application of these fluids in pipes are increasing, attention to fluid hammer phenomenon and its numerical modelling seems necessary. The present study aims to bridge the gap between fluid hammer theory and upper convected Maxwell fluids mechanics. For this purpose, at first, two equations representing the conservation of mass and momentum govern the transitional flow for non-Newtonian fluids are derived and then a two-step variant of the Lax-Friedrichs (LxF) method is used to discretize the governing equations of fluid hammer using Maxwell model relations. Computational results are presented in terms of the time history of pressure head at

critical points of a pipe such as at the valve and mid-length of the pipe. The results reveal that the viscoelastic fluid effects significantly contribute to attenuation time of transient flow. To our knowledge, this study represents the first attempt to the modelling of upper-convected-Maxwell fluid hammer due to the sudden closure of a downstream valve in a simple reservoir-pipe-valve system. The schematic shape of the problem is shown in Fig. 1.

Fig. 1 shows that the pipe system consists of a reservoir at the upstream end of the horizontal pipeline and a valve at the downstream end discharging to the atmosphere. This paper is organized as follows:

Section I: Transitional flows as Introduction section is defined and then the studies related to fluid-hammer phenomenon, is reviewed.

Section II: Firstly, the equations governing of the fluid hammer including continuity and momentum equations are presented and constitutive equations for modelling of upper-convected-Maxwell fluid are introduced and non-dimensionalized. Then LxF numerical method used is explained. As following, to assess the accuracy of the numerical method and proposed model, firstly, the governing equations of classical water- hammer are derived and then numerical solutions and Zielk method solutions for steady state at the valve and pipe midpoint are compared.

Section III: For obtaining the results near to the physical conditions, we used the properties of a dilute polymeric solution and a real geometry to obtain the typical dimensionless groups.

Section IV: The effect of Debora and Reynolds numbers as non-dimensional parameters on the behavior of upper-convected-Maxwell fluid during fluid hammer phenomena in a simple reservoir-pipe-valve system are investigated and compared to Newtonian fluid.

## 2- Formulation

### 2- 1- Governing equations

The equations for fluid transients in pipes are continuity and axial momentum equations. The continuity equation for transient pipe flow in a cylindrical coordinate system considering axial symmetry can generally be stated as follows [17]:

$$\frac{\partial H}{\partial t} + v_z \frac{\partial H}{\partial z} + \frac{c^2}{g} \left( \frac{1}{r} \frac{\partial (rv_r)}{\partial r} + \frac{\partial v_z}{\partial z} \right) = 0 \quad (1)$$

where  $v_z$  and  $v_r$  represent the axial and radial velocity components, respectively,  $c$  is the wave speed,  $H$  is the pressure head and  $t$  is time. The momentum equation in cylindrical coordinates in the axial direction is [17]:

$$\rho \left[ \frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} \right] + \frac{\partial p}{\partial z} - \left[ \frac{1}{r} \frac{\partial (r\tau_{rz})}{\partial r} \right] = 0 \quad (2)$$

where  $\rho$  is density,  $p$  is pressure,  $\tau_{rz}$  is stress shear in the liquid and:

$$c^2 = \frac{\frac{E_f}{\rho}}{1 + \frac{E_f D}{e E_p}} \quad (3)$$

Here,  $E_f$  is the bulk modulus of compressibility for the fluid,  $E_p$  is the Young's modulus of elasticity for the pipe material,  $e$  is the pipe thickness and  $D$  is the pipe diameter [18]. Using order of magnitude analysis, Wahba [19] and Ghidaoui et al. [1] showed that the nonlinear convective terms could be neglected from the continuity and axial momentum equations, since the wave speed is several orders of magnitude larger than the flow velocity. Moreover, it should be noted that the radial velocity in the laminar flow at the pipe wall and at the centerline is zero.

The problem modeling is done one-dimensionally, so for fluid hammer modeling with UCM fluid, using the above assumptions, the Eqs. (1) and (2) can be integrated across the pipe cross section and the transitional pipe flow model takes the following form:

$$\frac{\partial H}{\partial t} + \frac{c^2}{g} \frac{\partial \bar{V}}{\partial z} = 0 \quad (4)$$

$$\frac{\partial \bar{V}}{\partial t} + g \frac{\partial H}{\partial z} - \frac{2}{\rho R} \tau_{rz}|_{r=R} = 0 \quad (5)$$

where  $\bar{V}$  is the average cross-sectional velocity and  $R$  is the pipe radius.

$$\bar{V} = \frac{1}{A} \int v_z dA \quad (6)$$

## 2- 2- Constitutive equations

In this paper, upper convected Maxwell model is used as the constitutive equation. This model was proposed by Maxwell [20]. The general form of this model is as follows [17]:

$$\boldsymbol{\tau} + \lambda \overset{\nabla}{\boldsymbol{\tau}} = 2\eta \dot{\boldsymbol{\gamma}} \quad (7)$$

Where  $\boldsymbol{\tau}$  is the stress tensor,  $\lambda$  is the relaxation time,  $\eta$  is viscosity of the polymer,  $\overset{\nabla}{\boldsymbol{\tau}}$  is upper convected derivative of the stress tensor and defined as:

$$\overset{\nabla}{\boldsymbol{\tau}} = \frac{D\boldsymbol{\tau}}{Dt} - ((\nabla\mathbf{v})^T \cdot \boldsymbol{\tau} + \boldsymbol{\tau} \cdot (\nabla\mathbf{v})) \quad (8)$$

$\dot{\boldsymbol{\gamma}}$  is the upper convected derivatives of the rate of the strain tensor as follows:

$$\dot{\boldsymbol{\gamma}} = (\nabla\mathbf{v}) + (\nabla\mathbf{v})^T \quad (9)$$

Considering the Poiseuille velocity distribution profile equation for laminar flow in a long pipe and replacing the appropriate values in Eq. (7) the shear stress  $\tau_{rz}|_{r=R}$  is obtained. So, the governing equations for viscoelastic fluid hammer are given by:

$$\frac{\partial H}{\partial t} + \frac{c^2}{g} \frac{\partial \bar{V}}{\partial z} = 0 \quad (10)$$

$$\frac{\partial \bar{V}}{\partial t} + g \frac{\partial H}{\partial z} - \frac{2}{\rho R} \tau_{rz}|_{r=R} = 0 \quad (11)$$

$$\tau_{rz}|_{r=R} + \lambda \frac{\partial \tau_{rz}}{\partial t} = -\eta \frac{4\bar{V}}{R} \quad (12)$$

It should be noted in the above equations that convective terms have been neglected. Replacing relaxation time term in the momentum equation with zero, the classical water-hammer equations are obtained.

$$\frac{\partial H}{\partial t} + \frac{c^2}{g} \frac{\partial \bar{V}}{\partial z} = 0 \quad (13)$$

$$\frac{\partial \bar{V}}{\partial t} + g \frac{\partial H}{\partial z} + \frac{8\eta\bar{V}}{\rho R^2} = 0 \quad (14)$$

Replacing  $f = \frac{64\eta}{\rho \cdot 2R} = \frac{64}{Re}$  in Eq. (14) for laminar fluid hammer, the conventional form of the momentum equation in classical water hammer is obtained:

$$\frac{\partial \bar{V}}{\partial t} + g \frac{\partial H}{\partial z} + \frac{f\bar{V}|V|}{2D} = 0 \quad (15)$$

where  $f$  is the Darcy-Weisbach friction factor and  $g$  is the acceleration due to gravity.

## 2- 3- Initial and boundary condition

Initial and boundary conditions complete the mathematical description of the problem. The initial conditions are taken according to the steady state situation of the system. The boundary conditions describe the situation at the pipe ends, where for instance a reservoir or valve is located. The boundary conditions that describing a constant head reservoir with a pipe rigidly connected to it, is  $H = H_0$  where Subscript 0 shows the value of variables in steady state situation of the system.

## 2- 4- Non-dimensionalization of the equations

The different variables are normalized according to the following relations:

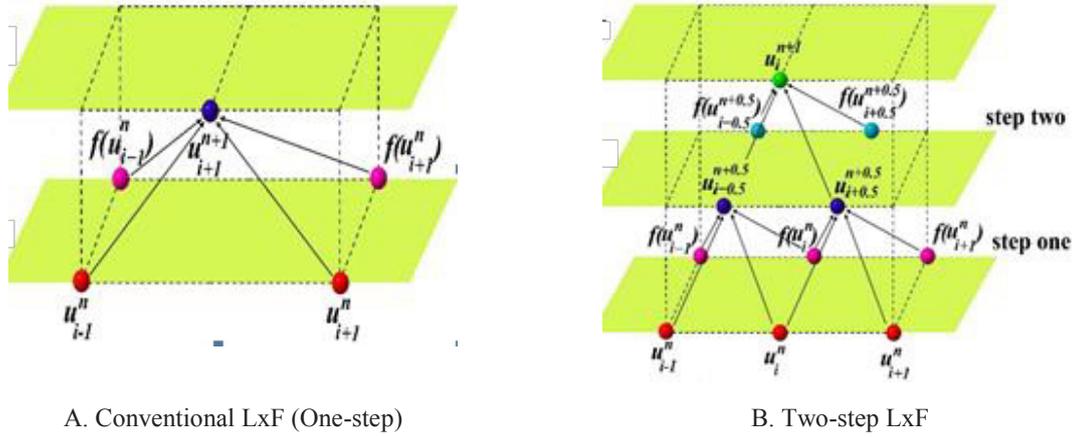


Fig. 2. Stencil of LxF method [22]

$$\bar{V}^* = \frac{\bar{V}}{v_0}, H^* = \frac{H}{cv_0/g}, z^* = \frac{z}{D},$$

$$t^* = \frac{t}{D/c}, \bar{\tau}^* = \frac{\bar{\tau}}{\rho cv_0}$$

(16)

where  $v_0$  is the velocity in steady state and non-dimensional numbers involved in this study are:

$$De = \frac{\lambda}{D/v_0}, M = \frac{v_0}{c}, Re = \frac{\rho v_0 D}{\eta_p + \eta_s}$$

(17)

where  $De$  denotes the Deborah number which is a dimensionless number defined based on the corresponding relaxation time of each viscoelastic medium [17],  $M$  is Mach number and is defined as the square root of the ratio of inertia force to the elastic force and  $Re$  is Reynolds number. Substituting into the Eqs. (10) to (12), the non-dimensional equations take the form:

$$\frac{\partial H^*}{\partial t^*} + \frac{\partial \bar{V}^*}{\partial z^*} = 0$$

(18)

$$\frac{\partial \bar{V}^*}{\partial t^*} + \frac{\partial H^*}{\partial z^*} - 4\bar{\tau}^*_{rz} = 0$$

(19)

$$\bar{\tau}^*_{rz} + \frac{De}{M} \cdot \frac{\partial \bar{\tau}^*_{rz}}{\partial t^*} = -8 \cdot \frac{\bar{V}^*}{Re} \cdot M$$

(20)

The non-dimensional equations for Newtonian fluid hammer are also as follows:

$$\frac{\partial H^*}{\partial t^*} + \frac{\partial \bar{V}^*}{\partial z^*} = 0$$

(21)

$$\frac{\partial \bar{V}^*}{\partial t^*} + \frac{\partial H^*}{\partial z^*} + \frac{32}{Re} \cdot M \bar{V}^* = 0$$

(22)

### 2- 5- LxF numerical method

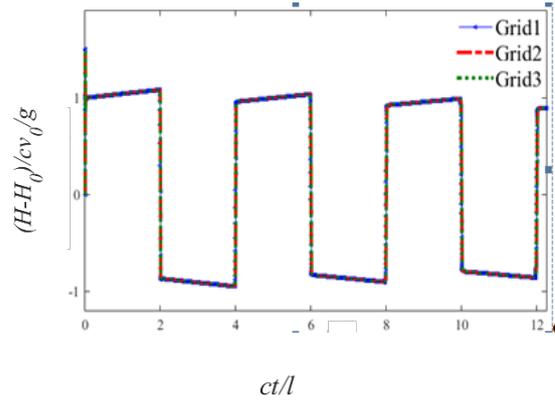
The basis of LxF method is the finite difference method and it's a good choice for solving Partial Differential Equations (PDEs). The LxF method is conservative and monotone, therefore, this is a Total Variation Diminishing (TVD) method [21]. As for the original Godunov method, the LxF scheme is based on a piecewise constant approximation of the solution, but it does not require solving a Riemann problem for time advancing and only uses flux estimates. LxF method is available for all forms of PDEs [21]. The stability condition is  $\frac{c\Delta t}{\Delta x} \leq 1$  and  $c$  is wave speed. Here, the numerical method used is a two-step variant of LxF method. Generally, multi-step methods increase convergence and accuracy in the numerical problems. In this study, two-step variant of LxF method is used. In this method, firstly, a half time step is taken based on LxF scheme on a staggered mesh. Next, the second half step is implemented based on LxF to reach at the solution on the original mesh. In Fig. 2, stencil of conventional LxF and two-step LxF are plotted.

Fig. 2 shows that in two-step LxF method, one time step is divided into two halves. Note that in this state, at the first step, the values of the function  $U$  in spatial nodes  $i=1, 2, \dots, v-1$  and the time  $n+0.5$  on a grid mesh is obtained where  $v$  is the node in the place of the valve and in the second half step, the values of the function  $U$  in spatial nodes  $i=1, 2, \dots, v-1$  and the time  $n+1$  on the original mesh is achieved.

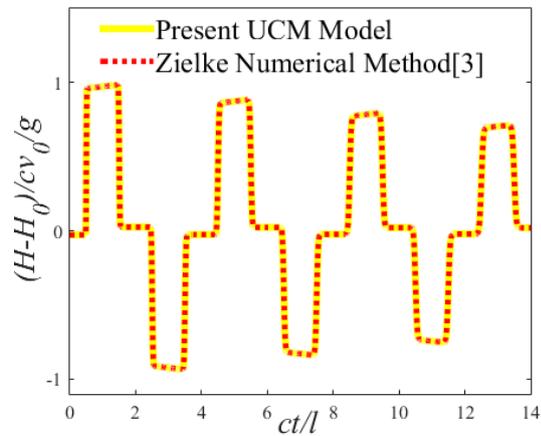
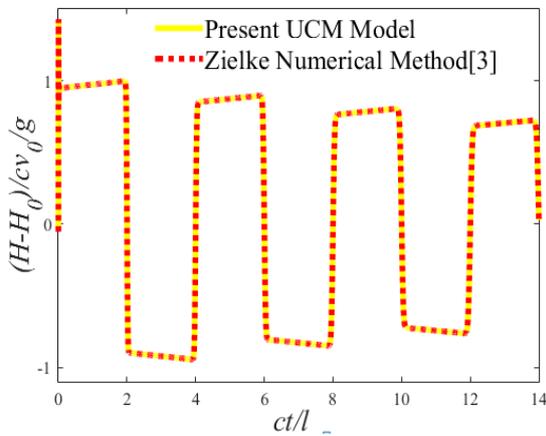
In this one-dimensional simulation, the stability condition is considered 0.99. Table 1 lists the spatial step sizes for the three different grids used in the verification of the present numerical procedure.

**Table 1. Non-dimensional spatial step sizes for the different grids**

Grid no.	$\frac{\Delta x}{l}$
1	0.00100
2	0.0005
3	0.00033



**Fig. 3. Grid independence for pressure-time history at the valve**



**Fig. 4. Validation of present UCM model for Newtonian fluid hammer**

Fig. 3 shows the pressure-time history at the downstream valve using all three grids. As can be seen from the figure, the medium and fine grids provide nearly identical results. In order to reduce computational cost, the second grid in this paper is used which in it, the pipe is divided into 2000 parts. To validate the proposed model, the results of the derived equations for the newtonian fluid are compared to Zielke [3] numerical method results for steady state friction on experimental data [23] at the valve and pipe midpoint.

Fig. 4 shows a good agreement between the results of an upper convected Maxwell model for Newtonian fluid and Zielke [3] numerical method results for steady state friction on experimental data in terms of the pressure-time history at the valve and pipe midpoint, respectively.

### 3- Numerical Modelling

In this section a polymer as an upper-convected-Maxwell fluid made in the laboratory is examined and the effect of Deborah and Reynolds numbers on the behavior of this polymer in different cases during fluid hammer phenomena due to the sudden closure of a downstream valve in a simple reservoir-pipe-valve system is studied. Firstly, for obtaining

the results near to the physical conditions, we used the properties of a dilute polymeric solution and a real geometry to obtain the typical dimensionless groups. For this purpose, a solution of polyacrylamide (100 ppm) in a glycerin/de-ionized water is considered as typical viscoelastic fluid. The molecular mass of polyacrylamide is  $M_w = 5 \times 10^6$  gr/mol and degree of purity of glycerin is 99%. The viscometric test of this solution indicate that the viscosity has a constant value of 0.08918 Pa.s in a wide range of shear rate [24] so it could be considered as a Boger liquid and the UCM constitutive equation is suitable to describe the mechanical behavior of this solution. The results of curve fitting of four modes generalized Maxwell model on the data of sweep frequency test at constant 10% of strain are presented in Table 2. Here, mode zero indicate the Newtonian contribution of model. Based on the data of this table, the fluid has an average relaxation time of 1.9 s [24].

So the value of relaxation time is considered 1.9 s. The other properties and pipe configuration data are presented in Table 3.

Generally in fluid hammer phenomenon, after the sudden closure of the valve, the fluid's velocity at the valve

**Table 2. Spectrum of relaxation time and viscosity [24]**

Mode no.	Dynamic viscosity (Pa.s)	Relaxation time constant (s)
0	0.0319	0
1	0.0625	7.088e-4
2	0.0131	0.2469
3	0.0025	10.2117
4	0.0151	9.9311

$$\bar{\lambda} = \frac{\sum_1^4 \eta_i \lambda_i}{\sum_1^4 \eta_i} = 1.9$$
**Table 3. Properties and pipe configuration data**

Properties	Values
Pipe length (m)	36.09
Mean velocity (m/s)	0.128
Pressure wave speed (m/s)	1324
Pipe diameter (m)	0.0253
Darcy-Weisbach friction factor	0.8
Specific density of fluid (kg/m <sup>3</sup> )	2200
Dynamic Viscosity (Pa.s)	0.08918
Reynolds Number	80
Deborah Number	9.6
Mach number	9.66e-5

reaches to zero and at the same moment, kinetic energy is completely transformed into the potential. This imposed potential energy causes the pressure head at the valve to rise equivalent to Joukowsky head. The Joukowsky pressure rise is the maximum pressure rise that would occur during the transient when viscous effects are neglected and is equal to  $\frac{c v_0}{g}$ . It should be noted that the all qualifications of numerical modeling such as diameter and length of pipe, wave speed,.. except fluid type are similar to experiment of Holmboe and Rouleau [23].

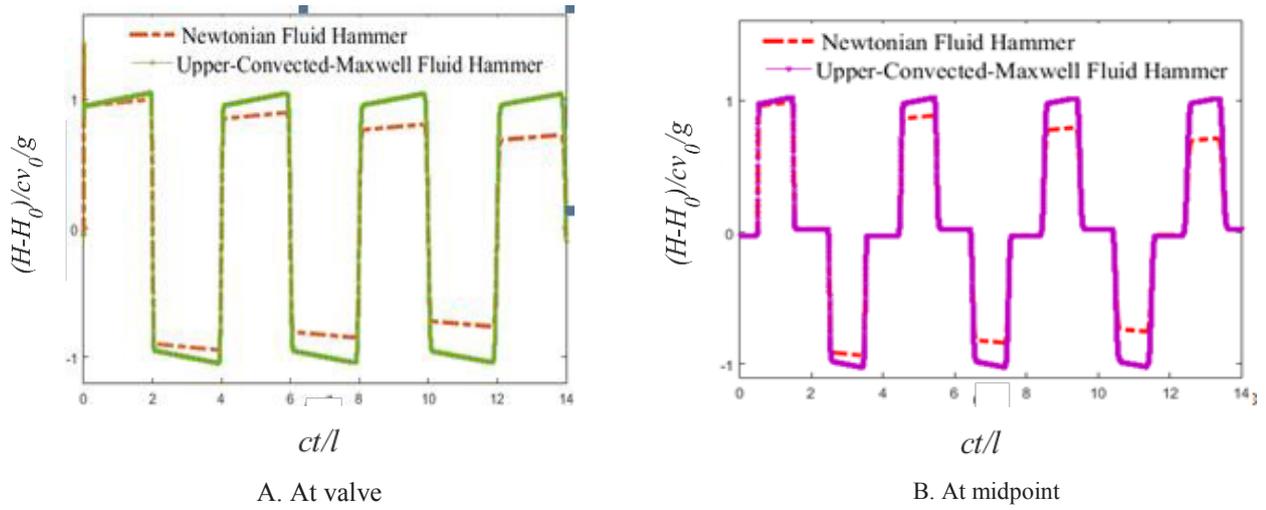
In Fig. 5 two key points can be observed. The first point is related to the maximum pressure at the valve. The Joukowsky pressure rise must be the maximum pressure rise that would occur during the transient at the valve when viscous effects are neglected, but in Fig. 5 the pressure rise is higher than Joukowsky's head, from  $t^*=0$  to  $t^*=2$  which this point can be observed in both Newtonian and upper convected Maxwell fluid. The second point is associated with attenuation time or damping time of transitional flow in two fluids. As shown in the Fig. 5, the height of the transitional flow in the upper convected Maxwell fluid compared to the Newtonian fluid is higher which leads to longer attenuation time.

The first point is related to pipeline packing or line packing phenomenon [18]. In this phenomenon, the value

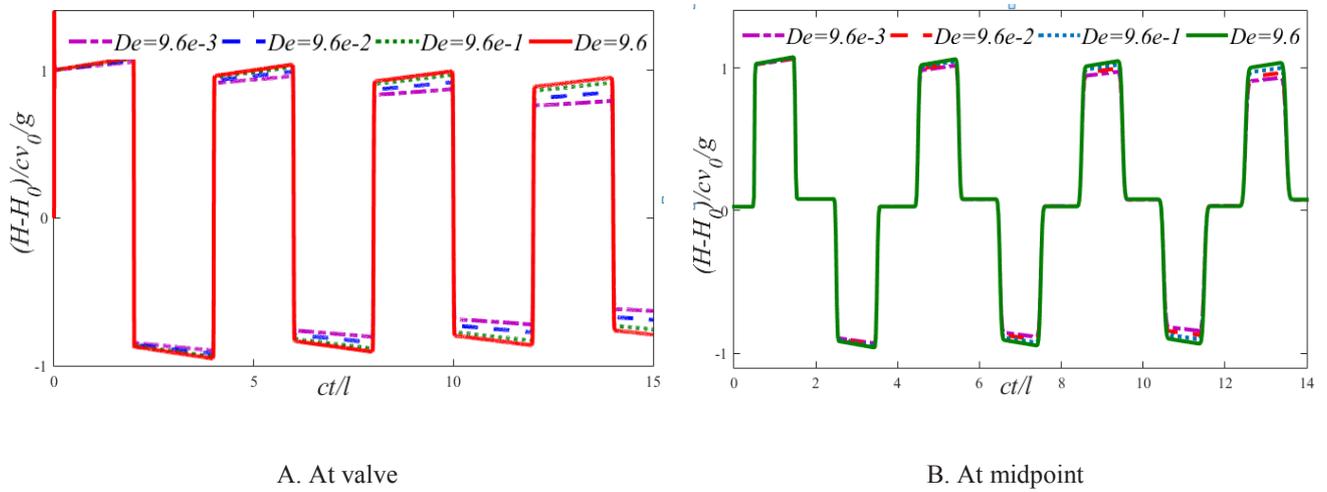
of transient pressure continues to rise above the Joukowsky pressure value due to frictional effects. In the case of the second point, it must be referred to the viscoelastic properties of the fluid. In a Newtonian fluid, after the imposition of the potential energy caused by the sudden closure of the valve, viscous characteristic of the liquid damps the pressure wave gradually. In an UCM fluid, solid and liquid properties of in it, show different reactions to this sudden potential energy at the same time. In fact, an UCM fluid, has elastic properties. Relaxation time as an elastic property plays an important role in storing the potential energy imposed on the fluid. This character of an UCM fluid, is caused the damping time of the transition flow becomes longer compared to Newtonian fluid.

#### 4- Results and Discussion

To study more precisely the pressure wave behavior, numerical modeling is also performed for other polymeric solutions with less molecular weight and relaxation time constant and the effect of dimensionless numbers of governing equation on pressure time history at valve and midpoint is investigated. It should be noted that Mach number in fluid hammer phenomenon is very little [1], therefore its effect is ignored.



**Fig. 5.** The comparison of pressure time history in Newtonian and upper convected Maxwell fluid hammer  
 $Re = 80, M = 9.66e - 5$



**Fig. 6.** The effect of Deborah number on pressure time history during upper convected Maxwell fluid hammer  
 $Re = 80, M = 9.66e - 5$

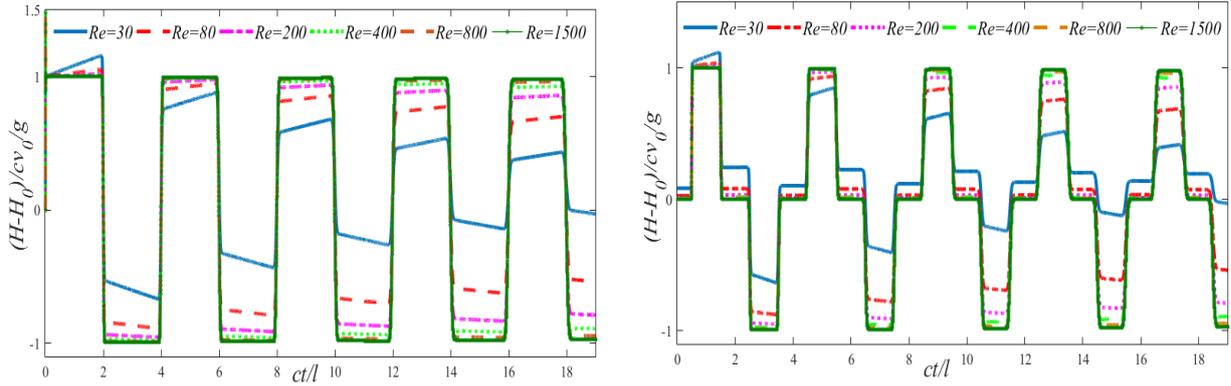


Fig. 7. The effect of Reynolds number on pressure time history in Newtonian Fluid hammer  $De = 0, M = 9.66e - 5$

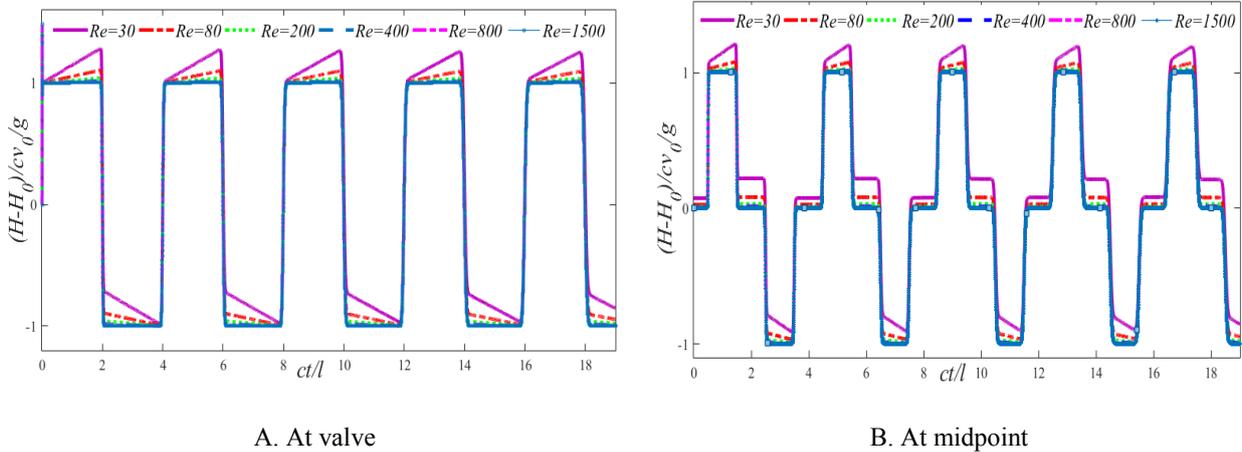


Fig. 8. The effect of Reynolds number on pressure time history in upper convected Maxwell fluid hammer  $De = 9.6, M = 9.66e - 5$

#### 4- 1- The effect of Deborah number

To investigate the effect of Deborah number on pressure time history at the valve and midpoint, the mentioned polymer with a number of polymers with less relaxation times which resulted in less Deborah numbers are selected. In Fig. 6 the effect of Deborah number on pressure time history during upper convected Maxwell fluid hammer.

According to Fig. 6 with increasing Deborah number, The height of the pressure wave increases. The reason for this behavior is related to relaxation time constant. This property supports the solid properties in UCM fluid. When the solid property is increased in the UCM fluid, the tendency to hold imposed energy is increased and so damping time of transitional flow becomes longer. In fact the behavior of polymer in the low Deborah number is similar to Newtonian fluid. It also can be observed that in all cases at valve, line packing or pipeline packing effect increases the pressure from the Joukowsky pressure.

#### 4- 2- The effect of Reynolds number

The pressure wave of fluid hammer, even in a Newtonian case, is sensitive to Reynolds number (Eq. (22)). In Figs. 7 and 8 the effect of Reynolds number on pressure time history in Newtonian and upper convected Maxwell fluid are shown.

As shown in Figs. 7 and 8, the overall pressure changes in Newtonian and UCM fluids are similar but the sensitivity is higher in the Newtonian case, especially for low Reynolds numbers. It means in laminar fluid hammer phenomenon, the strength of fluid viscosity as a friction effect in increased and consequently transient flow damping occurs sooner. In fact, the larger Reynolds number, the longer the attenuation time and this general trend repeats similarly in UCM fluid hammer. An important point in Figs. 7 and 8 is the intense impact of line packing or pipeline packing phenomenon [18] at valve. As it is clear, the lower Reynolds number, the higher the viscosity of fluid and thus line packing effect can be more noticeable. In low Reynolds, in Newtonian fluid, line packing

effect is observed only at the first time period but in UCM fluid the effect of this phenomenon continue to more time periods and damping occurs longer compared to Newtonian fluid.

### 5- Conclusion

A special kind of viscoelastic fluid hammer phenomenon called upper-convected-Maxwell fluid hammer is modeled. The fluid transient is generated by the sudden closure of the downstream valve. Equations representing the conservation of mass and momentum govern the transitional flow in the pipes. Upper-convected-Maxwell model relations are used as constitutive equations. The numerical method used is a two-step variant of LxF method. Dimensionless groups

derived at the governing equations during this phenomenon are Deborah, Reynolds and Mach which Reynolds and Mach numbers also can be derived at Newtonian state. Regardless of Mach number because of the very small amount, the effect of other non-dimensional groups on pressure time history during fluid hammer at valve and midpoint of the pipe was investigated and compared to the Newtonian fluid hammer. The results of this study show that relaxation time coefficient as one of the main characteristics of the UCM fluid plays an important role in increasing the height of the pressure wave. Also, the effect of this characteristic leads to the reduction of sensitivity to Reynolds number changes compared to Newtonian fluids.

### Nomenclature

$c$	Wave speed (m/s)
$D$	Pipe diameter (m)
$De$	Deborah number
$E$	Bulk modulus of compressibility (Pa)
$f$	Darcy-Weisbach friction factor
$e$	Pipe thickness (m)
$g$	Gravity acceleration (m/s <sup>2</sup> )

### Greek symbols

$\eta$	viscosity (Pa.s) Polymer p
$\tau$	Stress (Pa) Tensor
$\nabla$	
$\tau$	Upper convected derivative of the stress tensor (Pa)
$\rho$	Density (kg/m <sup>3</sup> )
$\bar{\tau}_{rz}$	Average stress components in the liquid in the corresponding surface and directions (Pa)
$\bar{\tau}_{zz}$	Average stress components in the liquid in the corresponding surface and directions (Pa)
$\delta$	The thickness of the boundary layer (m)
$\bar{\tau}$	Average stress
$\lambda$	Relaxation time (s)
$\nu$	Poisson's ratio

$H$	Pressure head (m)
$k$	The coefficient of restriction for axial pipe movement
$M$	Mach number
$P$	Pressure (Pa)
$R$	Pipe Radius (m)
$Re$	Reynolds number
$v_0$	Velocity in steady state (m/s)
$\bar{V}$	Average cross-sectional velocity (m/s)

### Superscripts

$n$	Previous time steps
$n+1$	Next time steps

### Subscripts

f	Fluid
p	Pipe material
r	Radial direction
z	Axial direction

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