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Nano-Scaled Plate Free Vibration Analysis by Nonlocal Integral Elasticity Theory

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ABSTRACT: In the current study, a finite element method is developed using the principle of total potential energy based on nonlocal integral elasticity theory to investigate the free vibration behavior of nano-scaled plates. The classical plate theory is considered for deriving the formulations of the plate. The eigenvalue problem is extracted by using the variational principle, and corresponding natural frequencies of free vibration are obtained using a numerical method. Different boundary conditions, various geometries, and kernel types can now be appropriately analyzed by using the nonlocal finite element method proposed in the current article. The results of the present study are compared with those available in the literature. Then the effects of nonlocal parameters, geometrical parameters, surface effects and various boundary conditions, including all sides clamped, all sides simply supported and clamped free, on the free vibration behavior of nano-scaled plates are investigated. It is concluded that considering the nonlocal effect results in a reduction of the natural frequencies. Natural frequencies decrease by increasing the length of the square plate and increasing the aspect ratio of the rectangular plate. Besides, it has been seen that variations of natural frequencies with the nonlocal parameter become less noticeable for plates with more considerable lengths.

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1- Introduction

In recent years, because of the beneficial mechanical, chemical and thermal specifications of nanomaterials, their applications are growing in many fields, such as mechanical engineering, electronics, biology and chemistry. Discovery of carbon nanotubes [1,2] and graphene sheets [3,4] have accelerated the increasing attention of scientific and industrial communities in nanostructures. The nanostructures have been efficiently used as nano-devices, nano-sensors, nano-actuators, atomic force microscopes, nano-composites etc. The mechanical behavior of nanostructures can be studied by experiments [5–7] and theoretical models that consist of atomistic models [8–12] and continuum mechanics models. In comparison with experimental and atomistic models, the continuum theories can be applied more conveniently. This is due to the fact that in the experimental investigations, the preparation conditions in nano-scale are sometimes difficult to achieve. Besides, the atomistic models are obviously more computationally demanding. Some of the size-dependent continuum theories which have been used for studying the small-sized structures are micro-polar theory [13], couple stress theory [14], micro-morphic theory [15] and nonlocal elasticity theory [16]. According to nonlocal elasticity theory [16], the stress at a given point is the function of strains at all points in the surrounding neighborhood. Two general forms exist for Eringen's nonlocal elasticity theory, i.e. integral and differential form. Even though the nonlocal differential theory is extracted from the nonlocal integral theory under certain conditions and as such it has some restrictions in facing more general problems, it has widely been used due to its simpler nature. Kitipornchai et al. [17] are among the first ones who have studied the vibration of nano-plates using a continuum mechanics model. Lu et al. [18] have used the nonlocal elasticity theory to study the bending and vibration of a simply supported rectangular nano-plate based on Kirchhoff and Mindlin plate theories. The governing equations have been achieved by integrating the equations of motion through the thickness. It is concluded that the nonlocal parameters have significant effects on the mechanical characteristics of nano-plates. Pradhan and Phadikar [19] have proposed an analytical method to study the natural vibration of single layered and double layered nano-plates based on nonlocal elasticity theory. The equations of motion are obtained for classical and first-order shear deformation theories. They have shown that the effect of nonlocal parameter is more considerable for smaller plates. Murmu and Pradhan [20] have investigated the free vibration of single-layered graphene sheets embedded in an elastic medium based on nonlocal elasticity theory. They have used the Differential Quadrature Method (DQM) to solve the problem numerically and, the effects of small-scale parameter, stiffness of the medium and aspect ratio on the natural frequencies are studied. Aghababaei and Reddy [21] have investigated the free vibration and bending behavior of a rectangular nano-plate considering the nonlocal elasticity theory. The formulations are based on the third-order shear deformation theory. The bending and vibration problems are solved analytically and the effects of nonlocal parameter on the bending deflection and vibration frequency are investigated. They have also compared the results with the first-order and classical plate theories. Ansari et al. [22] have analyzed the free vibration of simply supported and clamped single layered graphene sheets considering the nonlocal elasticity theory. The generalized DQM is used to numerically solve the governing equations and the results are compared with those of molecular dynamics. Simsek [23] has used the

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nonlocal elasticity theory to study the dynamic response of an elastic single-walled carbon nanotube subjected to a moving harmonic load based on Euler-Bernoulli beam theory considering simply-supported boundary conditions. Newmark's direct integration method and the modal analysis method has been used to obtain the time-domain responses and the effects of nonlocal parameter, velocity, excitation frequency and aspect ratio are investigated. Also the results have been validated by comparing the free vibration frequencies and static deflections of the Carbon NanoTube (CNT) with those available in the literature. Aydogdu [24] have presented a generalized beam theory to study bending, buckling and vibration of nano-scaled beams considering Eringen's nonlocal elasticity theory. Some special cases assuming various beam theories, including those of Euler-Bernoulli, Levinson, Reddy, Timoshenko and Aydogdu have been considered in the formulations. Also, the effects of geometrical and nonlocal parameters have been studied. Alibeigloo [25,26] has used the three-dimensional theory of elasticity based on nonlocal differential theory to study the vibration behavior of nano-plates. The vibration problem has been solved by utilizing the state-space method and Fourier series, and a closed-form solution has been obtained. Also, the effects of various parameters (e.g. nonlocal parameter, length of plate, thickness etc.) on the natural frequencies have been investigated. Karličić et al. [27] studied the buckling and free vibration of bonded multi-nano plates using the nonlocal elasticity theory. They have used Navier's method and trigonometric method to obtain the exact solution for the system of differential equations. Simsek [28] has proposed a beam model for nonlinear vibration of nano-beams based on the nonlocal elasticity theory. He employed the Hamilton's principle to derive the governing equations and related boundary conditions considering the von-Karman's non-linear strain-displacement relations and Euler-Bernoulli beam theory. Also the Galerkin method and He's variational method have been used to obtain an approximate analytical solution for nonlinear frequency of the nano-beam. The effect of nonlocal parameter on the nonlinear frequency ratios has been investigated for three different boundary conditions and it has been concluded that the effects of nonlocal parameter on the nonlinear responses of nano-beams are important and should be considered. Aydogdu [29] have used the nonlocal elasticity theory to develop an elastic rod model for studying the axial vibration of double-walled carbon nanotubes. The effects of small-scale parameter, van-der-Waals forces and geometrical parameters on the axial vibration have been investigated. It has been concluded that the nonlocal effects are more sensible for the rods with smaller length and the van-der-Waals forces have non-negligible effect on the vibration characteristics of the double-walled carbon nanotubes. Zhang et al. [30] have obtained the exact length-scale parameters for the vibration of initially stressed plate. For this purpose, they have presented a micro-structured beam-grid model for obtaining the vibration and buckling solutions and used the nonlocal plate model. Behera and Chakraverty [31] have employed the Rayleigh-Ritz method to study the free vibration behavior of non-homogeneous nano-plates based on nonlocal elasticity theory. They have investigated the effects of nonlocal parameter, material parameters and size-dependency on the frequency. Mehralian and Beni [32] have examined the vibration of Functionally Graded (FG) nanotube based on nonlocal strain gradient theory using the first order shear deformation shell model and the effects of material length scale, thickness ratio and length ratio on the frequency have been studied. Zeighampour and Beni [33] have investigated the wave propagation in fluid-conveying Double-Walled Carbon NanoTubes (DWCNTs) using shear-deformable shell model and the nonlocal strain gradient theory. The Hamilton's principle has been used to derive the governing equations and the effects of fluid velocity, foundation stiffness, wave number, nonlocal parameter and material length on the wave propagation have been studied. Zeighampour and Beni [34] also have used the nonlocal strain gradient principle to study the wave propagation in a composite laminated cylindrical microshell. Zeighampour and Beni [35] have used the nonlocal strain gradient theory to study the wave propagation in cylindrical viscoelastic Single-Walled Carbon NanoTubes (SWCNTs) surrounded by a medium of viscoelastic foundation. They derived the governing equations using the Hamilton's principle and Kelvin-Voigt viscoelastic model. In the aforementioned studies, the nonlocal differential elasticity (and strain gradient) theories are considered to investigate the mechanical behaviors of nano-structures. It is noted that the nonlocal differential elasticity theory, which is extracted from the more general integral form under some special conditions, faces some limitations. For instance, the nonlocal differential elasticity theory is obtained considering some special kernel functions, also the implementation of simply supported and free boundary conditions are ambiguous [36]. In order to overcome the latter restrictions, some researchers have conducted some studies based on nonlocal integral elasticity theory. Polizzotto [37] has extended the three variational principles, (i.e. the total potential energy, the Hu-Washizu and complementary energy principles), to the nonlocal integral elasticity theory. A suitable framework for applying the Finite Element Method (FEM) to nonlocal problems is theorized by using the principle of total potential energy. Pisano et al. [38] have used the FEM based on nonlocal integral elasticity to numerically analyze a plate under tension. They have discussed the computational issues of the method in detail. The method is verified and its effective application is approved. Taghizadeh et al. [39,40] have used the FEM considering the nonlocal integral elasticity to study the bending and buckling problems of a nano-scaled beam. They have compared the results with those of nonlocal differential elasticity theory and investigated the effects of different boundary conditions and nonlocal parameters on the bending deflection and buckling load. Naghinejad and Ovesy [41] have proposed a FEM to study the vibration behavior of nano-scaled beams based on nonlocal integral elasticity theory. The results are compared with those of nonlocal differential elasticity and the effects of nonlocal and geometric parameters on the free vibration of Euler-Bernoulli nano-scaled beams are investigated. Naghinejad and Ovesy [42] also have studied the free vibration of viscoelastic nano-scaled beams based on nonlocal integral elasticity theory and FEM. The effects of nonlocal parameter, viscoelastic parameter, geometrical parameter and boundary conditions on the natural frequencies have been investigated.

In the current study, a FEM is developed for analyzing the free vibration behavior of nano-scaled plates. The principle of total potential energy in conjunction with the nonlocal integral elasticity are used to provide the FEM formulations. The relations are based on Classical Plate Theory (CPT). Natural frequencies are then obtained by numerically solving the corresponding eigenvalue problems for nano-scaled plates. The results of the present study are compared with those of local elasticity and nonlocal differential elasticity theory, wherever available in the literature. In addition, the effects of nonlocal parameter, different boundary conditions, geometrical parameters and surface effects on the vibration behavior of nano-scaled plates are investigated. In comparison to the nonlocal differential elasticity theory, the present FEM is capable of modeling the various types of boundary conditions and different geometries of nano-plates more effectively. To the best of the author's knowledge it is the first time the FEM in conjunction with the nonlocal integral elasticity is used to study the vibration behavior of nano-plates, noting that the current method provides more versatility, especially in dealing with the boundary conditions.

2- Nonlocal Integral Elasticity Constitutive Equations

Using classical continuum mechanics, which ignores the small-scale effects, leads to inaccurate results in analyzing the nano-sized structures. So, for the appropriate modeling of the nanostructures, the size-dependent continuum theories ought to be used. One of the popular size-dependent theories is the nonlocal elasticity theory that has been developed by Eringen and Edelen [43,44]. In solid mechanics, an integral-type nonlocal material model is a model in which the constitutive law at a point of a continuum involves weighted averages of a state variable or of a thermodynamic force over a certain neighborhood of that point. Clearly, nonlocality is tantamount to an abandonment of the principle of local action of the classical continuum mechanics. A gradient-type nonlocal model, while adhering to this principle mathematically, takes the field in the immediate vicinity of the point into account by enriching the local constitutive relations with the first or higher gradients of some state variables or thermodynamic forces. A salient characteristic of both the integral- and gradient-type nonlocal models is the presence of a characteristic length or material length in the constitutive relation [45]. Nonlocal elasticity theory, in the general integral form, states that the constitutive law at a point is a function of weighted averages of state variables over a certain neighborhood [45]. Accordingly, for a linear isotropic continuum the nonlocal stress field $t(x)$ is expressed as

$$t(x)=\int_{V'}\alpha(|x-x'|,\tau)\sigma(x')dV' \tag{1}$$

where $\sigma(x)$ is the local stress tensor at point x and is given by Hooke law as

$$\sigma(x)=D:\varepsilon(x) \tag{2}$$

In which, D is the fourth-order elastic moduli tensor and ε is the local strain tensor. In Eq. (1), α is the nonlocal kernel, or attenuation function, which is a function of the Euclidean distance between x (reference point) and x' (all points in the vicinity), and $\tau=e_0a/l$ in which a and l are the internal and external characteristic length respectively, and e_0 is a specific constant for each material. By accommodating the dispersion curves of plane waves with those of experiments or atomic lattice dynamics, the shape of kernel function can be acquired for each material [16]. By increasing the distance from the reference point the value of kernel function i.e. the nonlocal effect is reduced and becomes negligible when $|x-x'|>L_R$, where L_R is called the effective distance. Using Eq. (2) another form of Eq. (1) becomes

$$t(x)=\int_{V'}\alpha(|x-x'|,\tau)D:\varepsilon(x')dV' \tag{3}$$

It is noted that if the internal length goes to zero, the nonlocal kernel would become Dirac delta in which case Eq. (3) would retrieve the classical local form (Hooke law). For this purpose the nonlocal kernel is required to satisfy the following normalization condition [37]

$$\int_{V_\infty}\alpha(|x-x'|,\tau)dV'=1 \tag{4}$$

where V_∞ is an infinite domain. Several studies have discussed the nonlocal kernel and its conditions near the boundaries. In the current study in line with that suggested by Pisano et al. [38] the kernel functions will be truncated near the boundaries of the plate (see Fig. 1).

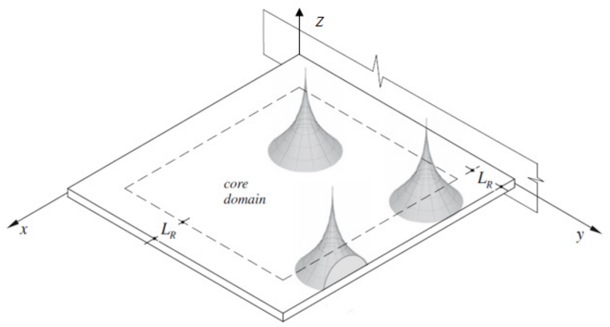

Fig. 1. Typical two dimension kernel function shapes for a plate [38]

Often in the literature [37], the constitutive law for the nonlocal integral elasticity is assumed as a two-phase material, in which phase one and two correspond to local and nonlocal elasticity respectively. The same assumption is adopted in the current study to have the constitutive equation as follows

$$t(x)=\zeta_1\sigma(x)+\zeta_2\int_{V'}\alpha(|x-x'|,\tau)\sigma(x')dV' \tag{5}$$

where ζ_1 and ζ_2 are the positive constants referring to the local and nonlocal volume fractions of the assumed two-phase material, respectively. It is considered that $\zeta_1+\zeta_2=1$. It is noted that by replacing the nonlocal kernel function α in Eq. (1) by its modified form $\bar{\alpha}$ as follows

$$\bar{\alpha}(|x'-x|,\tau)=\zeta_1\delta(x'-x)+\zeta_2\alpha(|x'-x|,\tau) \tag{6}$$

Eq. (5) can be obtained directly from Eq. (1).

3- Formulations of Nonlocal FEM based on Total Potential Energy Principle

In this section, a nonlocal FEM is developed to study the mechanical behavior of nano-scaled plates based on classical plate theory. As it is generally proposed by Polizzotto [37] the current nonlocal FEM is formulated by using the principle of total potential energy considering the nonlocal integral elasticity theory. Pisano et al. [38] and Taghizadeh et al. [40] have applied the aforementioned nonlocal FEM to tension and bending problems. Naghinejad and Ovesy [41] have extended the nonlocal FEM to study the vibration behavior of nano-scaled beams. The current study is dedicated to extend the nonlocal FEM for analyzing the free vibration behavior of the nano-scaled plates. One of the advantages of this method over the nonlocal differential elasticity ones is that, it is capable of accurately analyzing the simply supported and free boundary conditions. Besides, various geometries can be modeled using the current method.

By considering the nonlocal integral elasticity theory (Eq. (5)) and in the presence of inertia effects, the total potential energy is written as

$$\Pi(u)=\frac{1}{2}\left(\zeta_1\int_V \varepsilon(x):D:\varepsilon(x)dV+\zeta_2\int_V\int_V \alpha(|x'-x|,\tau)\varepsilon(x):D:\varepsilon(x')dV\,dV'\right)-W_{ext}-W_{inertia} \tag{7}$$

where works done by inertia and external forces are shown by $W_{inertia}$ and W_{ext} respectively. By applying the variations to minimize the total energy functional, Eq. (7) becomes

$$\begin{aligned}\delta\Pi(u)&=\zeta_1\int_V\delta\varepsilon(x):D:\varepsilon(x)dV\\&+\zeta_2\int_V\int_V\alpha(|x'-x|,\tau)\delta\varepsilon(x):D:\varepsilon(x')dV\,dV'\\&-\delta W_{ext}-\delta W_{inertia}=0\end{aligned} \tag{8}$$

Now, W_{ext} and $W_{inertia}$ are replaced by the corresponding terms,

$$\begin{aligned}\delta\Pi(u)&=\zeta_1\int_V\delta\varepsilon(x):D:\varepsilon(x)dV\\&+\zeta_2\int_V\int_V\alpha(|x'-x|,\tau)\delta\varepsilon(x):D:\varepsilon(x')dV\,dV'\\&-\int_V\bar{b}.\delta u\,dV-\int_{S_t}\bar{t}.\delta u\,dS-\int_V\delta u\left(-\rho\ddot{u}\right)dV=0\end{aligned} \tag{9}$$

where u is the displacement field, $\bar{t}$ and $\bar{b}$ are the surface force on S_t and body force in V respectively. Eq. (9) can be discretized and used as a foundation for the so called nonlocal finite element formulations. For this end, the total domain V is divided into N sub-domains V_n and the displacement field $u(x)$ and strain tensor $\varepsilon(x)$ of the n-th element are expressed as

$$u(x)=N_n(x)d_n \tag{10}$$

$$\varepsilon(x)=B_n(x)d_n \tag{11}$$

where $N_n(x)$ and $B_n(x)$ are the matrixes containing the shape functions and the corresponding partial derivatives of the shape functions of the n-th element, respectively. Matrixes N_n and B_n are defined considering employed element type. In addition, the nodal degrees of freedom for the n-th element are collected in vector d_n. Using Eqs. (10) and (11) and after the discretizing process, Eq. (9) becomes

$$\begin{aligned}\delta\Pi&=\zeta_1\sum_{n=1}^{N}\left(\delta d_n^T\left(\int_{V_n}B_n^T(x):D:B_n(x)dV\right)d_n\right)\\&+\zeta_2\sum_{n=1}^{N}\sum_{m=1}^{N}\left(\delta d_n^T\left(\int_{V_n}\int_{V_m}\alpha(|x'-x|,\tau)B_n^T(x):D:B_m(x')dV\,dV'\right)d_m\right)\\&-\sum_{n=1}^{N}\left(\delta d_n^T\int_{V_n}N_n^T(x)\bar{b}(x)dV\right)-\sum_{n=1}^{N}\left(\delta d_n^T\int_{S_{t_n}}N_n^T(x)\bar{t}(x)dS\right)\\&-\sum_{n=1}^{N}\left(\delta d_n^T\left(\int_{V_n}N_n^T(x).(-\rho N_n(x))dV\right)\ddot{d}_n\right)=0\end{aligned} \tag{12}$$

Now, the n-th element node displacement matrix d_n is connected to the global node displacement matrix U through the Boolean connectivity matrix C_n as

$$d_n=C_nU\quad\left(n\in\{1,\ldots,N\}\right) \tag{13}$$

By incorporating Eq. (13) into Eq. (12):

$$\begin{aligned}\delta\Pi&=\zeta_1\sum_{n=1}^{N}\left(\delta U^TC_n^T\left(\int_{V_n}B_n^T(x):D:B_n(x)dV\right)C_nU\right)\\&+\zeta_2\sum_{n=1}^{N}\sum_{m=1}^{N}\left(\delta U^TC_n^T\left(\int_{V_n}\int_{V_m}\alpha(|x'-x|,\tau)B_n^T(x):D:B_m(x')dV\,dV'\right)C_mU\right)\\&-\sum_{n=1}^{N}\left(\delta U^TC_n^T\int_{V_n}N_n^T(x)\bar{b}(x)dV\right)-\sum_{n=1}^{N}\left(\delta U^TC_n^T\int_{S_{t_n}}N_n^T(x)\bar{t}(x)dS\right)\\&-\sum_{n=1}^{N}\left(\delta U^TC_n^T\left(\int_{V_n}N_n^T(x).(-\rho N_n(x))dV\right)C_n\ddot{U}\right)=0\end{aligned} \tag{14}$$

The nodal load vector F_{Total}, total nonlocal stiffness matrix K^{NL}_{Total}, total local stiffness matrix K^{L}_{Total} and total mass matrix M_{Total} are assumed as

$$K^L_{Total}=\zeta_1\sum_{n=1}^{N}K_n^L \tag{15}$$

$$K^{NL}_{Total}=\zeta_2\sum_{n=1}^{N}\sum_{m=1}^{N}K^{NL}_{nm} \tag{16}$$

$$F_{Total}=\sum_{n=1}^{N}F_n^b+\sum_{n=1}^{N}F_n^t \tag{17}$$

$$M_{Total}=\sum_{n=1}^{N}M_n \tag{18}$$

where

$$K_n^L=C_n^T\left(\int_{V_n}B_n^T(x):D:B_n(x)dV\right)C_n \tag{19}$$

$$K_{nm}^{NL}=C_n^T\left(\int_{V_n}\int_{V_m}\alpha\left(\left|x'-x\right|,\tau\right)B_n^T\left(x\right):D:B_m\left(x'\right)dV\,dV'\right)C_m \quad (20)$$

$$F_n^b=C_n^T\int_{V_n}N_n^T\left(x\right)\bar{b}\left(x\right)dV \quad (21)$$

$$F_n^t=C_n^T\int_{S_{t_n}}N_n^T\left(x\right)\bar{t}\left(x\right)dS \quad (22)$$

$$M_n=C_n^T\left(\int_{V_n}N_n^T\left(x\right).\left(\rho N_n\left(x\right)\right)dV\right)C_n \quad (23)$$

It is noted that the total stiffness matrix consists of the nonlocal and local part (Eq. (24)).

$$K_{Total}=K_{Total}^{L}+K_{Total}^{NL} \quad (24)$$

It is noted that Eqs. (15) to (18) are known as the assembling process and Eqs. (19) to (23) are the relevant quantities of the elements. Furthermore, K_n^L is the local stiffness matrix of the n-th element and K_{nm}^{NL} is the nonlocal stiffness matrix demonstrating the nonlocal effects of the m-th element on the n-th element. By using Eqs. (15) to (24), Eq. (14) will be expressed as

$$\delta\Pi=\delta U^T K_{Total}U+\delta U^T M_{Total}\ddot{U}-\delta U^T F_{Total}=0 \quad (25)$$

Eq. (25) leads to the following form

$$K_{Total}U+M_{Total}\ddot{U}=F_{Total} \quad (26)$$

In the present study, the classical plate theory is used for analyzing the dynamic behavior of nano-scaled plates. For this purpose the following expressions are considered for the strains of the plate.

$$\varepsilon=\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}u_{0,x}\\ v_{0,y}\\ u_{0,y}+v_{0,x}\end{Bmatrix}-z\begin{Bmatrix}w_{,xx}\\ w_{,yy}\\ 2w_{,xy}\end{Bmatrix} \quad (27)$$

where ε_x, ε_y and γ_{xy} are longitudinal strain, transverse strain and in-plane shearing strain respectively and u, v and w are the deflections of the mid-plane in direction of x, y and z. Considering the pure bending conditions, strain tensor (Eq. (27)) is expressed as

$$\varepsilon\left(x\right)=\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=-z\begin{Bmatrix}w_{,xx}\\ w_{,yy}\\ 2w_{,xy}\end{Bmatrix}=B_n\left(x\right)d_n \quad (28)$$

The four-node Adini-Clough quadrilateral plate bending elements with three degrees of freedom in each node are considered. For this element type the nodal displacement vector d_n, containing 12 degrees of freedom for the n-th element, is

$$\boldsymbol{d}_n=\begin{bmatrix}w_1 & w_{1,x} & w_{1,y} & w_2 & w_{2,x} & w_{2,y} & w_3 & w_{3,x} & w_{3,y} & w_4 & w_{4,x} & w_{4,y}\end{bmatrix} \quad (29)$$

The shape functions of the considered element type in the local coordinate system are

$$N_n=\frac{1}{8}\begin{Bmatrix}e\left(a-\xi-\eta\right)\\ e\left(1-\xi^2\right)\\ e\left(1-\eta^2\right)\\ f\left(a+\xi-\eta\right)\\ -f\left(1-\xi^2\right)\\ f\left(1-\eta^2\right)\\ g\left(a+\xi+\eta\right)\\ -g\left(1-\xi^2\right)\\ -g\left(1-\eta^2\right)\\ h\left(a-\xi+\eta\right)\\ h\left(1-\xi^2\right)\\ -h\left(1-\eta^2\right)\end{Bmatrix} \quad (30)$$

In Eq. (30):

$$\begin{aligned}&e=\left(1-\xi\right)\left(1-\eta\right),f=\left(1+\xi\right)\left(1-\eta\right),g=\left(1+\xi\right)\left(1+\eta\right),\\ &h=\left(1-\xi\right)\left(1+\eta\right),a=2-\xi^2-\eta^2\end{aligned} \quad (31)$$

Finally for the plate element, the stiffness and mass matrix would be obtained using the following equations. Also, the Gauss-Legendre quadrature rule will be used for evaluating the numerical integrations. It is noted that in the following equations the local coordinates are used.

$$K_n^L=C_n^T\left(\sum_{j=1}^{NINT}\sum_{i=1}^{NINT}\left(\int_{h_n}B_n^T:D:B_n w_i w_j\left(\det J\right)dz\right)_{\xi_i,\eta_j}\right)C_n \quad (32)$$

$$\begin{aligned}K_{nm}^{NL}=C_n^T(&\sum_{j=1}^{NINT}\sum_{i=1}^{NINT}\sum_{j'=1}^{NINT}\sum_{i'=1}^{NINT}\int_{h_n}\int_{h_m}\alpha\left(\left|x_{\xi_{i'},\eta_{j'}}^{'}-x_{\xi_i,\eta_j}\right|,\tau\right)\\ &*\left(B_n^T\right)_{\xi_i,\eta_j}:D:\left(B_m\right)_{\xi_{i'},\eta_{j'}}w_i w_j w_i w_j\left(\det J'\right)_{\xi_{i'},\eta_{j'}}\left(\det J\right)_{\xi_i,\eta_j}dzdz)C_m\end{aligned} \quad (33)$$

$$M_n=C_n^T\left(\sum_{j=1}^{NINT}\sum_{i=1}^{NINT}\left(\int_{h_n}N_n^T.\left(\rho N_n\right)w_i w_j\left(\det J\right)dz\right)_{\xi_i,\eta_j}\right)C_n \quad (34)$$

where $NINT$ is the number of Gauss points in each direction, h_n is the thickness of the element, w_i and w_j are the weights of the integration and J is the Jacobian matrix. As it is seen, different types of kernel function could be used in the presented FEM. For comparing the results of the current study with those of the nonlocal differential elasticity, which is obtained by considering some special kernel types such as the one presented as Eq. (35), the same two dimension kernel type will be assumed to obtain the results

$$\alpha\left(\left|x-x'\right|,\tau\right)=\lambda_0\left(2\pi l^2\tau^2\right)^{-1}K_0\left(\frac{\left|x-x'\right|}{l\tau}\right) \tag{35}$$

K_0 is the modified Bessel function of the second kind, $\tau=e_0a/l$ and λ_0 is a normalization factor expressed as

$$\lambda_0=\frac{1}{\int_{V_\infty}\left(2\pi l^2\tau^2\right)^{-1}K_0\left(\left|x-x'\right|/l\tau\right)dV} \tag{36}$$

In addition because of the importance of the surface effects in nano-scale [46–51] , also the following relations are added to the formulation to consider the surface effects. For this purpose using the procedure explained above and as it has been expressed in [47,52], the stiffness matrix corresponding to surface effects is considered as follows:

$$Ks_n^L=C_n^T\left(\int_{V_n}B_{sn}^T\left(x\right):Cs:B_{sn}\left(x\right)dA\right)C_n \tag{37}$$

$$Ks_{nm}^{NL}=C_n^T\left(\int_{V_n}\int_{V_m}\alpha\left(\left|x'-x\right|,\tau\right)B_{sn}^T\left(x\right):Cs:B_{sm}\left(x'\right)dA'dA\right)C_m \tag{38}$$

where Ks_n^L is the local surface effect stiffness matrix and Ks_{nm}^{NL} is the nonlocal surface effect matrix. Also Cs is the surface elastic constants matrix which is expressed as:

$$Cs=h/2\begin{bmatrix}2\mu_s+\lambda_s & \tau_s+\lambda_s & 0\\ \tau_s+\lambda_s & 2\mu_s+\lambda_s & 0\\ 0 & 0 & \frac{1}{2}\left(2\mu_s-\tau_s\right)\end{bmatrix} \tag{39}$$

In which τ_s, μ_s and λ_s are the residual surface tension and the surface Lame constants respectively. Also B_s is the matrix containing the derivatives of the shape functions as [52]:

$$B_s=\begin{bmatrix}0 & -\frac{\partial N_1}{\partial x} & 0 & \dots\\ \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & \dots\\ \frac{\partial N_1}{\partial x} & -\frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial x} & \dots\end{bmatrix} \tag{40}$$

It is noted that, as it is clear from the procedure of the current method and using the principle of total potential energy to extract the nonlocal finite element formulations, any kind of boundary conditions can be analyzed by the current method, i.e. one of the main advantages of the FEM is that various boundary conditions, e.g. clamped, simply supported, free, partial boundary conditions, etc. can be handled. So, the natural boundary conditions can be achieved with no specific difficulty if the current nonlocal integral FEM is applied. It is noted that the latter boundary conditions involve some certain complications in the case of nonlocal differential elasticity methods. The latter difference between two methods arises from the fact that, only the essential (geometric) boundary conditions should be satisfied in the FEM.

4- Governing Equations and Vibration Analysis

For analyzing the free vibration characteristics, the plate is under no body or surface forces. So, the right-hand side of Eq. (26) is zero.

$$(K_{Total}+K_s)U+M_{Total}\ddot{U}=0 \tag{41}$$

The solutions of the form

$$U=A\left(x\right)e^{i\omega t} \tag{42}$$

are considered to evaluate the natural frequencies and mode shapes for the free vibration of the nano-plate. Inserting Eq. (42) in Eq. (41) yields

$$\left(K_{Total}+K_s-M_{Total}\omega^2\right)A=0 \tag{43}$$

Eq. (43) is an eigenvalue problem equation, which is used to obtain the natural frequencies and mode shapes of the free vibration. Eq. (43) has a non-trivial solution when the determinant of the following matrix is zero,

$$\left|K_{Total}+K_s-M_{Total}\omega^2\right|=0 \tag{44}$$

The total stiffness matrix K_{Total}, surface effects stiffness matrix C_s and total mass matrix M_{Total} of the nano-plate are achieved by the developed FEM. Then the system of equations, which is obtained by applying the condition of Eq. (44), is numerically solved and the eigenvalues (natural frequencies ω_n) and the eigenvectors (mode shapes A_n) are acquired. The numerical method which has been used for obtaining the eigenvalues is the well-known Jacobi eigenvalue algorithm. The Jacobi eigenvalue method repeatedly performs rotations until the matrix becomes diagonal, then the elements in the diagonal are approximations of the eigenvalues. It is clear that, as it has been mentioned earlier, the current method is capable of modeling different types of boundary conditions. To this purpose, the essential boundary conditions are taken into account in the stiffness and mass matrixes, and the corresponding terms will be modified.

5- Results and Discussion

In the current article, the free vibration behavior of nano-scaled plates is going to be studied using the developed FEM based on nonlocal integral elasticity considering the classical plate theory. In this section, first the convergence study is carried out to determine the sufficient number of mesh grids for the study. Then, the natural frequencies obtained by the current study method are compared with those of nonlocal differential elasticity theory available in the literature (with and without surface effects). At last, the effects of nonlocal parameter, geometrical parameters and various boundary conditions on the free vibration of nano-scaled plates are investigated (without surface effects). The properties of the analyzed nano-plate are shown in Table 1.

Table 1. Properties of the nano-plate [17]

E [TPa]	v	h [nm]	l [nm]	ρ [kg/m^3]	e_0a [nm]
1	0.16	0.34	10	2250	0, 0.5, 1, 1.5, 2

It is noted that, wherever the non-dimensional frequency term is used it means that the frequency has been non-dimensionalized as the following form.

$$\bar{\omega}_{ND} = \omega l^2 \sqrt{\rho h / D},\ D = Eh^3 / 12\left(1-\nu^2\right)$$

The convergence study for a square nano-plate considering the nonlocal parameter e_0a=1 [nm] and all sides simply supported boundary conditions is carried out by studying the fundamental natural frequencies (see Table 2). Square elements of the same size are used for meshing the square plate. As it is seen for the mesh sizes of the 45×45 elements the results become significantly convergent. So for calculating the rest of the results, in the current paper, the mesh size of 45×45 elements have been used.

Table 2. Fundamental natural frequency of all-sides simply supported nano-plate based on nonlocal FEM

Number of square elements	Fundamental natural frequency [GHz] (e_0a =1 [nm])
3×3	71.5887
7×7	59.9211
11×11	55.953
15×15	54.1543
19×19	53.1874
23×23	52.6066
27×27	52.2302
31×31	51.9721
33×33	51.8128
37×37	51.789
41×41	51.788
45×45	51.788

Table 3 shows the comparison of non-dimensional natural frequencies of present analysis based on the nonlocal FEM with those of Pradhan and Phadikar [19], Murmu and Pradhan [20] and Ansari et al. [22] based on nonlocal differential elasticity theory for two kinds of boundary conditions and nonlocal parameters e_0a=0 and 1. It is noted that the results of Pradhan and Phadikar [19] are obtained using the Navier's solution and classical plate theory and Murmu and Pradhan [20] are obtained using the DQM and CPT whilst those of Ansari et al. [22] are obtained according to first order shear deformation theory and also molecular dynamics model considering zigzag and armchair Single-Layer Graphene Sheet (SLGS). As it is seen, there is a good agreement between the results for the local case (e_0a=0), but when the nonlocal effects are taken into account the results of the present study based on nonlocal integral elasticity tend to be smaller than those of the nonlocal differential elasticity. Besides, the discrepancy between the results are more pronounced for the case of simply supported boundary conditions in comparison with the clamped boundaries. It might be due to the fact that, the nonlocal differential elasticity theories have some difficulties satisfying the natural boundary conditions. In addition, the nonlocal differential equations are extracted from the relations of nonlocal integral elasticity in the regions far from the boundaries. However, the general agreement between the results is satisfactory.

In Table 4 the results of the current study considering the surface effects are compared with those of Karimi et al. [47] based on differential nonlocal elasticity and Two Variable Refined Plate Theory (TVRPT). The properties of the surface layer are considered as τ_s=0.83 N/m, μ_s=0.47 N/m and λ_s=0.28 N/m [47]. A fairly good agreement is seen between the results. Also it is seen that by increasing the length to thickness ratio the non-dimensional natural frequency increases.

The effects of the geometrical and nonlocal parameters on the free vibration behavior of nano-scaled plates considering different boundary conditions are to be studied. The properties of the nano-plate are given in Table 1. Variations of natural frequencies with nonlocal parameter are shown in Figs. 2 to 4 for different lengths of a square plate. Three different boundary conditions including all-sides simply supported, all-sides clamped and clamped-free boundaries have been considered. As it is seen, the natural frequencies decrease by increasing the nonlocal parameter (e_0a). The latter softening phenomenon is stronger for the smaller lengths of the plate, i.e. the variations of natural frequency with the nonlocal parameter are less noticeable for the plates with larger lengths. This behavior might be attributed to the importance of the nonlocal effects near the boundaries. Thus, for the square nano-plate of the smaller length the boundary conditions play a significant role on the nonlocal characteristics of the plate and the effects of nonlocal parameter on the natural frequencies will become

Table 3. Fundamental natural frequency comparison for a nano-plate

Boundary conditions	$\omega_{ND} = \omega l^2\sqrt{\rho h/D}$									
	Present	Pradhan and Phadikar [19]	Murmu and Pradhan [20]	Ansari et al. [22]	Present	Pradhan and Phadikar [19]	Murmu and Pradhan [20]	Ansari et al. [22]	Ansari et al. [22] - MD (zigzag)	Ansari et al. [22] - MD (armchair)
	e_0a = 0 [nm]				e_0a = 1 [nm]					
All-sides simply supported	19.71	19.72	19.73	20.65	15.52	18.02	18.01	18.88	17.62	17.84
All-sides clamped	35.90	-	-	37.65	28.16	-	-	33.78	34.36	34.84

Table 4. Fundamental natural frequency comparison considering the surface effects for a simply supported nano-plate (l=10 nm)

Length to thickness ratio	$\omega_{ND} = \omega l^2\sqrt{\rho h/D}$					
	Present [without surface effects]	Present	Karimi et al. [46]	Present [without surface effects]	Present	Karimi et al. [46]
	$e_0a = 0$ [nm]			$e_0a = 1$ [nm]		
l/h=10	19.71	29.82	29.88	15.52	24.43	28.77
l/h=5	19.71	24.85	19.44	15.52	20.85	18.04
l/h=2	19.71	19.87	12.54	15.52	15.74	11.48

more noticeable. In addition, the increase in plates length has effectively resulted in the reduction of nonlocal effects (e_0a/l) when assuming a constant e_0a value, because the effects of boundary conditions will diminish.

Figs. 5 to 7 show the variations of fundamental natural frequencies with length of the square plate for three boundary conditions. The results have been depicted for various nonlocal parameter values e_0a=0,0.5,1,1.5,2. As it is seen by increasing the length of the plate the natural frequencies will decrease regardless of the nonlocal parameter value. In addition, the natural frequency of the all-sides clamped plate has experienced more reduction compared to those of the plate with other boundary conditions. It is due to the fact that as it has been explained, the truncation of kernels near the boundaries results in the effect of nonlocality to be more pronounced there. Thus, for a stronger boundary condition this effect will be more prominent. For further comparison, Fig. 8 shows the variations of natural frequencies with length of the square plate for two boundary conditions and different nonlocal parameters in a single diagram. It is observed that by increasing the nonlocal parameter, the reduction of natural frequency with length of the plate decreases.

The effects of aspect ratio on the free vibration of a rectangular nano-plate are depicted in Figs. 9 to 11. Three different boundary conditions have been considered. As it is observed, by increasing the aspect ratio of the plate (length/width) the natural frequencies decrease. Moreover, for smaller aspect ratios the effects of nonlocal parameter are more pronounced in comparison with the larger aspect ratios. It is also seen that for a clamped free boundary condition increasing the aspect ratio results in the steeper decrease of the natural frequency in comparison with other two boundary conditions. It is due to the fact that by increasing the aspect ratio, i.e. length (b) to width (a) ratio, of the plate the length of the free edges increases and it leads to lower natural frequencies.

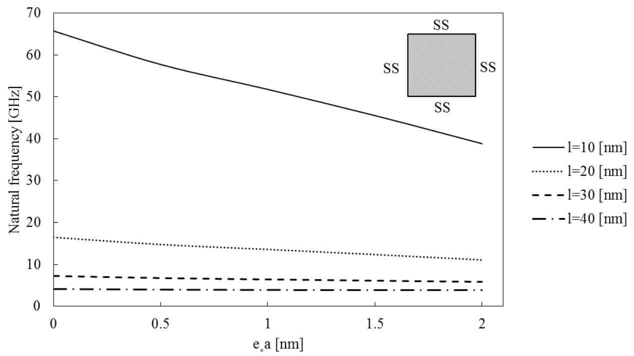

Fig. 2. Effects of nonlocal parameter (e_0a) on the fundamental natural frequencies of a square nano-plate with all-sides simply supported boundary conditions for different lengths

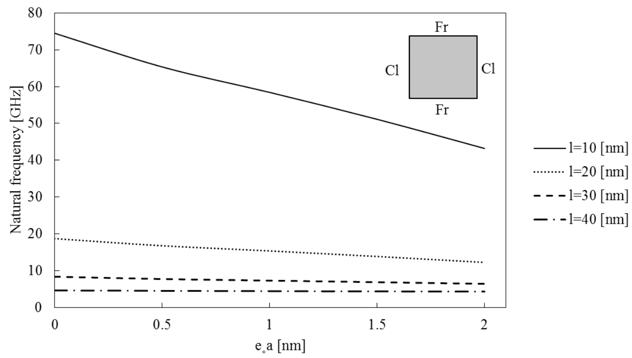

Fig. 4. Effects of nonlocal parameter (e_0a) on the fundamental natural frequencies of a square nano-plate with clamped-free boundary conditions for different lengths

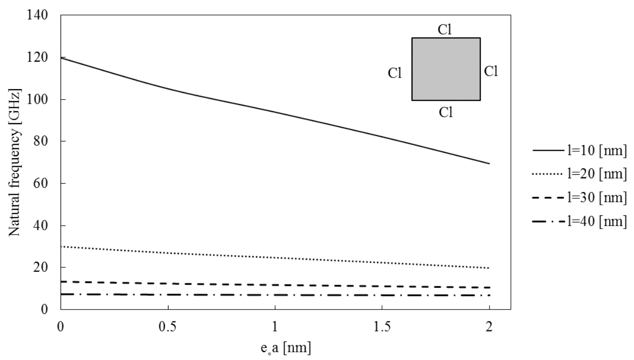

Fig. 3. Effects of nonlocal parameter (e_0a) on the fundamental natural frequencies of a square nano-plate with all-sides clamped boundary conditions for different lengths

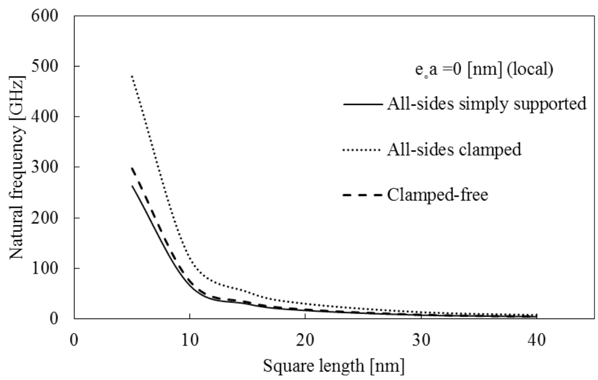

Fig. 5. Variations of fundamental natural frequencies with length of the square nano-plate considering e_0a=0 for different boundary conditions

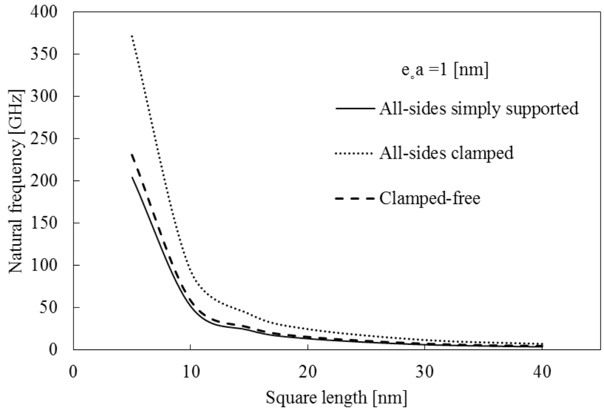

Fig. 6. Variations of fundamental natural frequencies with length of the square nano-plate considering e_0a=1 for different boundary conditions

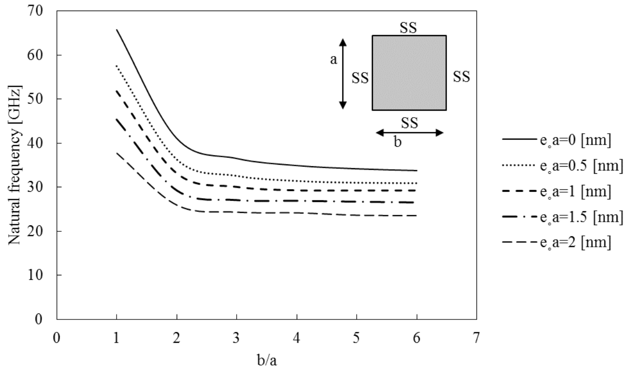

Fig. 9. Variations of fundamental natural frequencies with aspect ratio of a rectangular nano-plate considering all-sides simply supported boundary condition

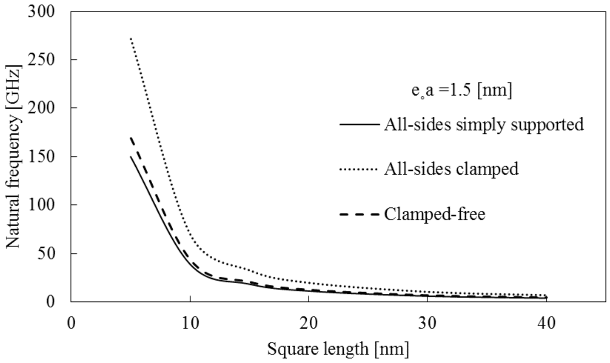

Fig. 7. Variations of fundamental natural frequencies with length of the square nano-plate considering e_0a=2 for different boundary conditions

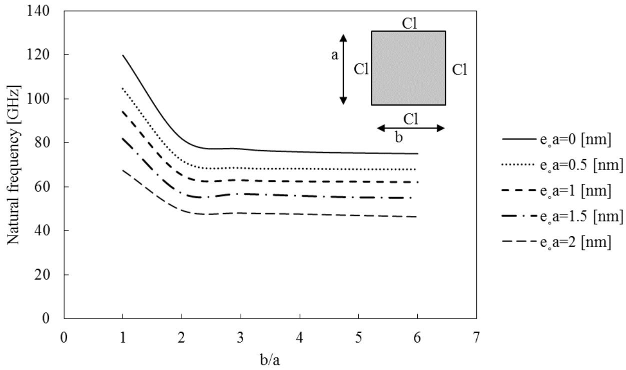

Fig. 10. Variations of fundamental natural frequencies with aspect ratio of a rectangular nano-plate considering all-sides clamped boundary condition

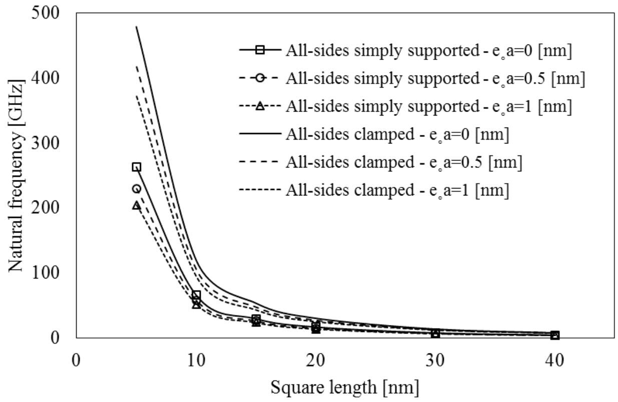

Fig. 8. Variations of fundamental natural frequencies with length of the square nano-plate considering different nonlocal parameters for clamped and simply supported boundary conditions

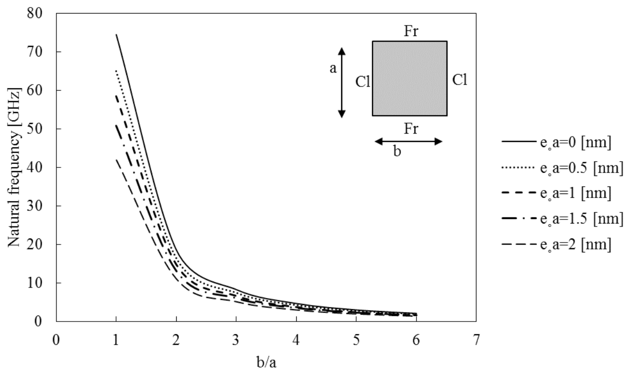

Fig. 11. Variations of fundamental natural frequencies with aspect ratio of a rectangular nano-plate considering clamped-free boundary condition

Figs. 12 to 14 show the variations of non-dimensional natural frequencies with length to thickness ratio of square plate for different nonlocal parameters. All-sides clamped, all-sides simply supported and clamped-free boundary conditions have been studied. It is seen that for a case of e_0a=0 (local analysis) the non-dimensional natural frequency is not affected by the length to thickness ratio. For a non-zero values of e_0a, i.e. nonlocal cases, the non-dimensional natural frequencies increase by increasing the length to thickness ratio. Also, the increase of nonlocal parameters causes the non-dimensional natural frequencies to decrease.

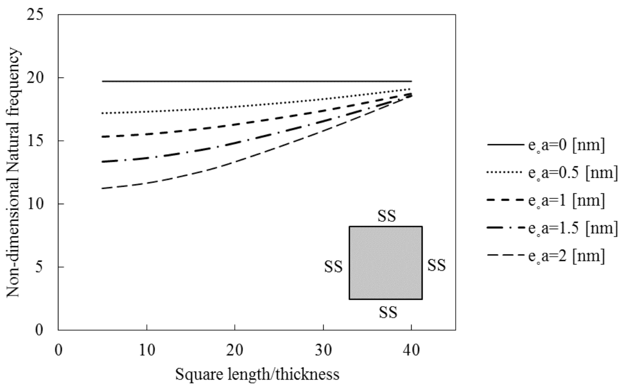

Fig. 12. Variations of non-dimensional natural frequencies with length to thickness ratio of square nano-plate for all-sides simply supported boundary conditions

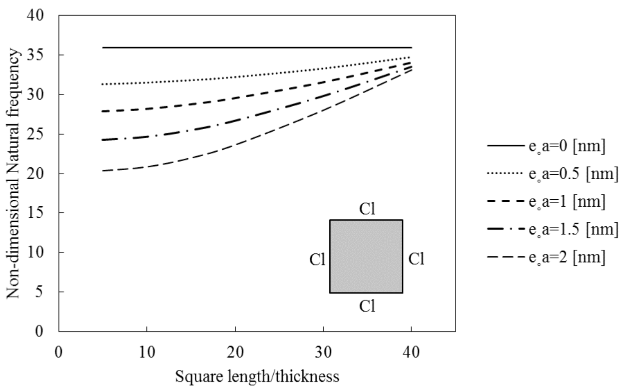

Fig. 13. Variations of non-dimensional natural frequencies with length to thickness ratio of square nano-plate for all-sides clamped boundary conditions

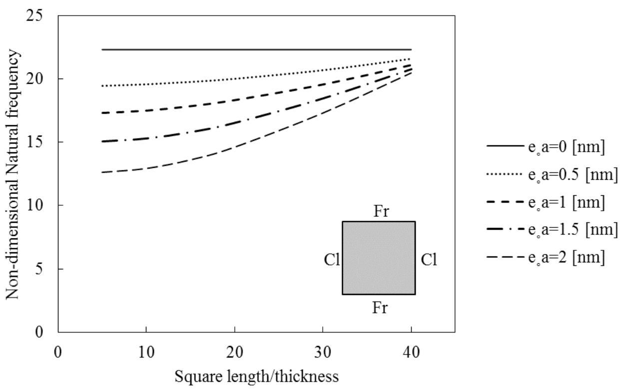

Fig. 14. Variations of non-dimensional natural frequencies with length to thickness ratio of square nano-plate for clamped-free boundary conditions

6- Conclusion

Based on the nonlocal integral elasticity a FEM is developed to study the dynamic properties of nano-scaled plates. The formulations are derived using the principle of total potential energy and the classical plate theory is considered. The natural frequencies are obtained by numerically solving the eigenvalue problems. Despite the difficulty of handling the simply supported and free boundary conditions in nonlocal differential elasticity theory, the current method is able to analyze various boundary conditions properly. In addition, different geometries and nonlocal kernel types can be considered. The results of the current study have been compared with those available in the literature, and the effects of nonlocal parameter, boundary conditions, geometrical parameters and surface effects on the free vibration of nano-scaled plates have been studied. It has been seen that, increasing the nonlocal parameter (e_0a) results in the natural frequencies to be decreased. Also, the natural frequencies will decrease by increasing the length of the square plate. For the plates with larger lengths, the variations of natural frequency with the nonlocal parameter are less noticeable. Increasing the aspect ratio leads to decreasing the natural frequencies of the plate. In addition, the effects of nonlocal parameter are more pronounced for smaller aspect ratios. Considering the surface effects by increasing the length to thickness ratio, the non-dimensional natural frequency increases.

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