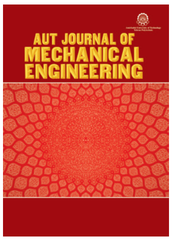

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Further Evaluation of Squeezing Flow and Heat Transfer of non-Newtonian Fluid with Nanoparticles Conveyed through Vertical Parallel Plates

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ABSTRACT: In this paper, the study of squeezing flow of sodium alginate (SA) a non-Newtonian fluid whose rate of shear is not constant with viscosity flows through a medium transporting nanoparticles of silver (Ag) and Alumina (Al_2O_3). The flow medium is a flat parallel plate arranged vertically against each other under steady flow condition. As the flow process arising from the mechanics can be described by ordinary nonlinear differential equation, the Adomian decomposition method been an effective, yet simple method is adopted to analyze the non-linear differential equation. This is used to investigate effect of squeezing flow and heat transfer on the nanofluid. Analytical results reported graphically depicts the effect of squeezing flow on heat transfer utilizing silver nanoparticles shows decreasing temperature distribution for plates coming together while as plates moves apart temperature distribution decreases further. Similar trend is observed adopting the alumina nanoparticle. However the silver nanoparticle having better thermal properties compared with alumina demonstrates higher heat transfer rate due to effect of varying fluid kinematic viscosity on heat exchange. Results generated from the study when compared with existing literature are in good agreement. Therefore study proves a good emphasis for the improvement of sodium alginate transport in biomedical, pharmaceuticals, manufacturing and chemical processes amongst others.

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1- Introduction

The increasing relevance of the sodium alginate, a non-Newtonian fluid in modern day science is due to its wide practical use. Applications including biomedical, pharmaceuticals and manufacturing. Vast effort has been devoted by scientist and engineers to this study owing to influence of rate of shearing on fluid viscosity. The physical phenomena of the sodium alginate in engineering process such as paper production, tissue formation engineering, textile and pharmaceuticals are described by constitutive relations which are nonlinear and of higher order.

In earlier effort to illustrate flow process, Ganji and Kachapi [1] analyzed nonlinear equations in fluids, using numerical and analytical schemes for providing solutions to differential equations while Ziabakhsh and Domairry [2] investigated the effects of natural convection on the flow of a non-Newtonian fluid between two vertical parallel plates. Also the heat and mass transfer effect of unsteady squeezing flow was presented by Mustafa et al. [3] between two parallel plates. Siddiqui et al. [4] presented the unsteady viscous squeezing fluid flow between parallel plates in the presence of magnetic force field .The thermodynamic analysis of third grade fluid was studied by Adesanya and Falade [5] were they presented the rate of entropy generation of the fluid flowing between horizontal parallel plates saturated with porous materials. Hoshyar et al. [6] adopted the homotopy analysis method to study the unsteady incompressible fluid flow between parallel plates. The effect of variable viscosity on fluid flow between parallel plates with shear heating was demonstrated by Myers et al. [7]. Kargar and Akbarzade [8] investigated non-Newtonian fluid flow between two infinitely long vertical parallel plates using both analytical and numerical approach.

Thermal conductivity of base fluids such as water, glycol and oils have been considerable improved upon the addition of nanometer sized metallic particles. As this enhances it thermal conductivity to about three times its state. Considerably improving the overall transport energy of the base fluid. Making it potentially useful in processes including fuel cells, microelectronic process and medicine. Creative approach such as this has been widely adopted by researchers in the investigation and study of flow and heat transfer properties of fluid [9-22].

In other to analyze the governing equations of the sodium alginate which is of an higher order. Numerical or analytical methods of solution must be utilised. Widely adopted analytical method of solution amongst researchers include the Homotopy Perturbation Method (HPM), the Variational Iteration Method (VIM), Perturbation Method (PM), the Homotopy Analysis Method (HAM) and the Differential Transformation Method (DTM) [23-40]. Methods such as HPM,VIM,DTM and HAM requires the use of computational stencils in handling large solution parameters resulting to large computational cost and time, due to need to find initial condition to satisfy boundary condition [41]. The classical perturbation technique (PM) is limited owing its weak nonlinearity problem and artificial perturbation parameter. The Adomian Decomposition Method (ADM) which involves decomposing nonlinear system of equations into linear and nonlinear terms makes the ADM a powerful,yet simplistic approach widely adopted by researchers. As it is not limited by an initial or guess term, round off errors, linearization or

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descritization.
From past literatures, no study as considered a comparative analysis of squeezing flow of the nanofluid with sodium alginate as base fluid. Therefore this current paper aims at investigating and providing a comparative analysis of the squeezing flow effect on the non-Newtonian sodium alginate adopting nanoparticles of silver (Ag) and Alumina (Al_2O_3). The ADM is employed as the suitable method of analysis.

2- Model Development and Analytical Solution

A non-Newtonian sodium alginate flows steadily under natural convection between two plates placed at a distance 2b vertically against each other. The walls are held at constant temperature but opposite in magnitude as illustrated in the problems model, Fig. 1. This is such that temperature difference causes a rise and fall of fluid near the wall. The formulation of the model development of the two component mix is developed following the assumptions that the fluid is incompressible, nanoparticles and fluid are in thermal equilibrium to each other and constant wall temperature. As illustrated from the boundary condition, plates and nanofluid are at equal velocity connoting the no slip condition. While constant temperature but magnitude difference leads to nanofluid rise near the left plate and fall near the right. Following the model proposed by [8,11] introducing the squeezing flow and nanofluid parameters.

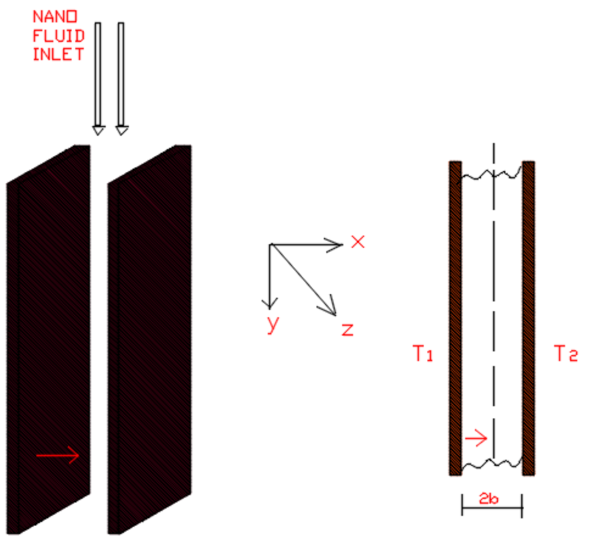

Fig. 1. Physical model of the problem

The governing equation based on the assumption above can be reduced to ordinary pairs of differential equation. This is presented as:

$$\mu\frac{d^2v}{dx^2}+6\beta_3\left(\frac{dv}{dx}\right)^2\frac{d^2v}{dx^2}+\rho_f\gamma\left(T-T_m\right)g=0 \tag{1}$$

$$\mu\left(\frac{dv}{dx}\right)^2+2\beta_3\left(\frac{dv}{dx}\right)^4+k\frac{d^2\theta}{dx^2}=0 \tag{2}$$

where the dimensionless parameters take the forms:

$$\delta=\frac{6\beta_3v_0^2}{\mu_f G^2},A_1=\frac{\rho_{nf}}{\rho_f},A_2=\frac{\mu_{nf}}{\mu_f},A_3=\frac{\left(\rho C_p\right)_{nf}}{\left(\rho C_p\right)_f},$$
$$E_c=\frac{\rho_f v_0^2}{(\rho C_p)_f(T_1-T_2)},\Pr=\frac{\mu_f(\rho C_p)_f}{\rho_f k_f}, \tag{3}$$
$$\theta=\frac{T-T_m}{T_1-T_2},S=\frac{\alpha b^2}{2\upsilon},V=\frac{v}{V_0}$$

With the aid of non-dimensional parameter Eq. (3). The Eqs. (1) and (2) are expressed as:

$$\frac{d^2V}{dX^2}+6S\,\delta A_1\left(1-\phi\right)^{2.5}\left(\frac{d^2V}{dX^2}\right)\left(\frac{d^2V}{dX^2}\right)^2+\theta \tag{4}$$

$$\frac{d^2\theta}{dX^2}+Ec.P\,\mathrm{rS}\left(\frac{A_2}{A_3}\right)\left(\left(1-\phi\right)^{-2.5}\right)\left(\frac{d^2V}{dX^2}\right)^2$$
$$+2\delta.Ec.\Pr.\left(\frac{1}{A_3}\right)\left(\frac{dV}{dX}\right)^4=0 \tag{5}$$

where the effective density ρ_{nf}, effective dynamic viscosity μ_{nf}, heat capacitance$(\rho C_p)_{nf}$ and thermal conductivity k_{nf} of the nanofluid are defined as follows [40]:

$$\rho_{nf}=\left(1-\phi\right)\rho_f+\phi\rho_s \tag{6}$$

$$\left(\rho C_p\right)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_s \tag{7}$$

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}} \tag{8}$$

$$A_1=\frac{k_{nf}}{k_f}=\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f-\phi(k_f-k_s)} \tag{9}$$

Taking the boundary condition as

$$v=0,\theta=0.5$$
$$v=0,\theta=-0.5 \tag{10}$$

Table 1. Thermo physical properties of sodium alginate, silver and alumina nanoparticle [10]

	Density (kg/m³)	Specific heat capacity (J/kg.K)	Thermal conductivity (W/m.K)
Sodium Alginate (SA)	989	4175	0.637
Silver (Ag)	10500	235	429
Alumina (Al_2O_3)	3970	765	40

The preferred analytical scheme, which is the ADM. Is adopted for providing approximate solutions to the ordinary differential, nonlinear, second order equations. Upon the application of the ADM to the Eqs. (3) and (4), the governing equation may be expressed as:

$$L_{xx}(\mathrm{v}) = -6\delta SA_1(1-\phi)^{2.5}\left(\frac{dv}{dx}\right)^2\frac{d^2v}{dx^2}+\theta \tag{11}$$

$$L_{xx}(\theta) = -Ec\,\mathrm{PrS}\left(\frac{A_2}{A_3}\right)(1-\phi)^{-2.5}\left(\frac{dv}{dx}\right)^2 -2\delta Ec\,\mathrm{Pr}\left(\frac{1}{A_3}\right)\left(\frac{dv}{dx}\right)^4 \tag{12}$$

To simplify the integrations, the differential operator of the highest order was taken as $L_{xx}=d^2/dx^2$ for the coupled equation.Thus inverting L_{xx} gives L_{xx}^{-1} . On applying L_{xx}^{-1} to Eqs. (11) and (12) gives

$$v = L_{xx}^{-1}\left[-6\delta SA_1(1-\phi)^{2.5}\left(\frac{dv}{dx}\right)^2\frac{d^2v}{dx^2}\right] -L_{xx}^{-1}\theta + C_1x + C_2 \tag{13}$$

$$\theta = L_{xx}^{-1}\left[-Ec\,\mathrm{PrS}\left(\frac{A_2}{A_3}\right)(1-\phi)^{-2.5}\left(\frac{dv}{dx}\right)^2\right] -L_{xx}^{-1}\left[2\delta Ec\,\mathrm{Pr}\left(\frac{1}{A_3}\right)\left(\frac{dv}{dx}\right)^4\right]+C_1x+C_2 \tag{14}$$

Utilizing the ADM, velocity and temperature may be given as

$$v = \sum_{n=0}^{\infty} v_n \tag{15-a}$$

$$\theta = \sum_{n=0}^{\infty}\theta_n \tag{15-b}$$

The nonlinear terms will be expressed in the form of Γ_n and Λ_n in the Adomian polynomials which yields

$$\sum_{n=0}^{\infty}\Gamma = 6\delta SA_1(1-\phi)^{2.5}\left(\frac{d}{dx}\sum_{n=0}^{\infty}v^2\right)\left(\frac{d^2}{dx^2}\sum_{n=0}^{\infty}v\right) \tag{16}$$

$$\sum_{n=0}^{\infty}\Lambda = Ec\,\mathrm{PrS}\left(\frac{A_2}{A_3}\right)(1-\phi)^{-2.5}\left(\frac{d}{dx}\sum_{n=0}^{\infty}v^2\right)\left(\frac{d^2}{dx^2}\sum_{n=0}^{\infty}v^4\right) \tag{17}$$

Utilising Eqs. (16) and (17) the Eqs. (13) and (14) may be expressed as

$$v = -L_{xx}^{-1}\theta - L_{xx}\left(\sum_{n=0}^{\infty}\Gamma\right)+C_1x+C_2 \tag{18}$$

$$\theta = -L_{xx}\left(\sum_{n=0}^{\infty}\Lambda\right)+C_1x+C_2 \tag{19}$$

where boundary conditions is expressed as

$$\sum_{n=0}^{\infty}v_n = 0, \sum_{n=0}^{\infty}\theta_n = 0.5 \qquad \sum_{n=0}^{\infty}v_n = 0, \sum_{n=0}^{\infty}\theta_n = -0.5 \tag{20}$$

The zeroth order can be obtained from the recursive relations Eqs. (13) and (14)

$$v_0 = C_1x + C_2 - L_{xx}^{-1}\theta_0 \tag{21}$$

$$\theta_0 = C_1x + C_2 + 0 \tag{22}$$

With leading order boundary condition expressed as

$$v_0 = 0, \theta_0 = 0.5 \qquad v_0 = 0, \theta_0 = -0.5 \tag{23}$$

The remaining order of the solutions is given as

$$v_{j+1} = L_{xx}^{-1}(\Gamma_j), \quad j \ge 0 \tag{24}$$

$$\theta_{j+1} = L_{xx}^{-1}(\Lambda_j), \quad j \ge 0 \tag{25}$$

With boundary condition expressed as

$$\sum_{n=0}^{\infty}v_n = 0, \sum_{n=0}^{\infty}\theta_n = 0.5 \qquad \sum_{n=0}^{\infty}v_n = 0, \sum_{n=0}^{\infty}\theta_n = 0.5 \tag{26}$$

From Eq. (16) polynomians of Adomian, Γ_n can be expressed as

$$\Gamma_0 = 6\delta SA_1(1-\phi)^{2.5}\left(\left(\frac{dv_0}{dx}\right)^2\frac{d^2v_0}{dx^2}\right) \tag{27}$$

$$\Gamma_1 = 6\delta SA_1(1-\phi)^{2.5}\left(\left(\frac{dv_0}{dx}\right)^2\frac{d^2v_1}{dx^2}+2\frac{dv_0}{dx}\frac{dv_1}{dx}\frac{d^2v_0}{dx^2}\right) \tag{28}$$

From Eq. (16) the Adomian polynomians, Λ_n can be obtained as

$$\Lambda_0 = Ec\,\mathrm{PrS}\left(\frac{A_2}{A_3}\right)(1-\phi)^{-2.5}\left(\frac{dv_0}{dx}\right)^2 +2\delta Ec\,\mathrm{Pr}\left(\frac{1}{A_3}\right)\left(\frac{dv_0}{dx}\right)^4 \tag{29}$$

$$\Lambda_1 = 2Ec\,\mathrm{PrS}\left(\frac{A_2}{A_3}\right)(1-\phi)^{-2.5}\frac{dv_0}{dx}\frac{dv_1}{dx} + 4\delta Ec\,\mathrm{Pr}\left(\frac{1}{A_3}\right)\left(\frac{dv_0}{dx}\right)^3\frac{dv_1}{dx} \tag{30}$$

Solution of zeroth order is obtained by simplifying the recursive relation Eqs. (21) and (22) using the zeroth order boundary condition Eq. (23) which yields.

$$v_0 = \frac{x^3}{4} - \frac{x}{4} \tag{31}$$

$$\theta_0 = -\frac{x}{2} \tag{32}$$

First order solution can be derived from Eqs. (27) and (29) which is expressed as

$$v_1 = L_{xx}^{-1}(\Gamma_0) \tag{33}$$

$$\theta_1 = L_{xx}^{-1}(\Lambda_0) \tag{34}$$

With the first order boundary condition given as

$$\begin{aligned} v_1 &= 0, \theta_1 = 0.5 \\ v_1 &= 0, \theta_1 = -0.5 \end{aligned} \tag{35}$$

Upon simplifying Eqs. (33) and (34) with the aid of first order boundary condition Eq. (35).This can be easily shown as

$$v_1 = 6\delta S A_1(1-\phi)^{2.5}\left(\frac{9x^5}{80} - \frac{3x^3}{48} + \frac{x}{160}\right) \tag{36}$$

$$\begin{aligned} \theta_1 &= Ec\,\mathrm{PrS}\left(\frac{A_2}{A_3}\right)(1-\phi)^{-2.5}\left(\frac{9x^6}{480} - \frac{3x^4}{96} + \frac{x^2}{32} + \frac{9x}{160} - \frac{3}{40}\right) \\ &+2\delta Ec\,\mathrm{Pr}\left(\frac{1}{A_3}\right)^* \\ &\left(\frac{81x^{10}}{23040} - \frac{27x^8}{3584} + \frac{27x^6}{3840} - \frac{3x^4}{768} + \frac{x^2}{512} + \frac{57x}{17920} - \frac{19}{4480}\right) \end{aligned} \tag{37}$$

Second order solution can be obtained from Eqs. (27) and (29) which is expressed as

$$v_2 = L_{xx}^{-1}(\Gamma_1) \tag{38}$$

$$\theta_2 = L_{xx}^{-1}(\Lambda_1) \tag{39}$$

With the second order boundary condition as follows

$$\begin{aligned} v_2 &= 0, \theta_2 = 0.5 \\ v_2 &= 0, \theta_2 = -0.5 \end{aligned} \tag{40}$$

Upon simplifying Eqs. (38) and (39) with the aid of second order boundary condition Eq. (40) can be easily shown as

$$v_2 = 6\delta S A_1(1-\phi)^{2.5}\left(\frac{135x^9}{18432} - \frac{183x^7}{10752} + \frac{329x^5}{25600} - \frac{33x^3}{7680} + \frac{57x}{50000}\right) \tag{41}$$

$$\begin{aligned} \theta_2 &= 2Ec\,\mathrm{PrS}\left(\frac{A_2}{A_3}\right)(1-\phi)^{-2.5}\left(\frac{15x^8}{7168} - \frac{13x^6}{3840} + \frac{33x^4}{7680} + \frac{x^2}{1280} + \frac{567x}{50000} - \frac{189}{12500}\right) \\ &+4\delta Ec\,\mathrm{Pr}\left(\frac{1}{A_3}\right)^* \\ &\left(\frac{135x^{12}}{270336} - \frac{297x^{10}}{184320} + \frac{531x^8}{286720} - \frac{161x^6}{153600} + \frac{9x^4}{122880} + \frac{9x}{12500} - \frac{3}{3125}\right) \end{aligned} \tag{42}$$

The summations of Eqs. (31) and (36) and (41) gives the ADM solutions for the velocity profile while Eqs. (32) and (37) and (42) gives the solution for temperature profile. Which is expressed as

$$\begin{aligned} v &= \frac{x^3}{4} - \frac{x}{4} + 6\delta S A_1(1-\phi)^{2.5}\left(\frac{9x^5}{80} - \frac{3x^3}{48} + \frac{x}{160}\right) \\ &+6\delta S A_1(1-\phi)^{2.5}\left(\frac{135x^9}{18432} - \frac{183x^7}{10752} + \frac{329x^5}{25600} - \frac{33x^3}{7680} + \frac{57x}{50000}\right) \end{aligned} \tag{43}$$

$$\begin{aligned} \theta &= -\frac{x}{2} + Ec\,\mathrm{PrS}\left(\frac{A_2}{A_3}\right)(1-\phi)^{-2.5}\left(\frac{9x^6}{480} - \frac{3x^4}{96} + \frac{x^2}{32} + \frac{9x}{160} - \frac{3}{40}\right) \\ &+2\delta Ec\,\mathrm{Pr}\left(\frac{1}{A_3}\right)^* \\ &\left(\frac{81x^{10}}{23040} - \frac{27x^8}{3584} + \frac{27x^6}{3840} - \frac{3x^4}{768} + \frac{x^2}{512} + \frac{57x}{17920} - \frac{19}{4480}\right) \\ &+2Ec\,\mathrm{PrS}\left(\frac{A_2}{A_3}\right)(1-\phi)^{-2.5}\left(\frac{15x^8}{7168} - \frac{13x^6}{3840} + \frac{33x^4}{7680} + \frac{x^2}{1280} + \frac{567x}{50000} - \frac{189}{12500}\right) \\ &+4\delta Ec\,\mathrm{Pr}\left(\frac{1}{A_3}\right)^* \\ &\left(\frac{135x^{12}}{270336} - \frac{297x^{10}}{184320} + \frac{531x^8}{286720} - \frac{161x^6}{153600} + \frac{9x^4}{122880} + \frac{9x}{12500} - \frac{3}{3125}\right) \end{aligned} \tag{44}$$

3- Results and Discussion

The results obtained from the analytical solutions are discussed here. Effect of rheological fluid parameters on squeezing flow and heat transfer are reported graphically in Figs. 2 to 10.The effect of silver as nanoparticle conveyed through the base fluid is considered in Figs. 2 to 7 whereas alumina is considered in Figs. 8 to 10. The Tables 2 and 3 expresses the validity of the approximate analytical method in providing solutions to the nonlinear problem. The solutions reported by Kargar and Akbarzade [8] for the simplified case without squeeze flow and nanoparticles using HPM and Runge-Kutta numerical solution are compared to the analytical solution obtained adopting the ADM. This proves to be in satisfactory agreement, showing the relevance of the ADM in providing solution to complex problems. The non-Newtonian parameter (δ) effect on the velocity profile is observed from Fig. 2 which shows increasing (δ) tends to increase shear leading to corresponding decrease in boundary layer thickness. This results to slight decrease in velocity distribution.

Table 2. Comparison of various values of X for velocity profiles. When $Pr=Ec=\delta=S=1$, $\phi=0.0$.

X	$V(X)$			Error	
	NS [8]	HPM[8]	Present work	HPM	Present work
-1.0000	0.000	0.0000	0.0000	0.0000	0.0000
-0.8000	0.0231	0.0239	0.0233	0.0008	0.0002
-0.6000	0.0314	0.0322	0.0316	0.0008	0.0002
-0.4000	0.0277	0.0284	0.0279	0.0007	0.0002
-0.200	0.0167	0.0166	0.0165	0.0001	0.0002
0.2000	-0.0147	-0.0151	-0.0150	0.0004	0.0003
0.4000	-0.0265	-0.0271	-0.0268	0.0006	0.0003
0.6000	-0.0305	-0.0312	-0.0308	0.0007	0.0003
0.8000	-0.0226	-0.0234	-0.0229	0.0008	0.0003
1.0000	-0.0000	0.00000	-0.0000	0.0000	0.0000

Table 3. Comparison of various values of X for temperature profiles. When $Pr=Ec=\delta=S=1$, $\phi=0.0$.

X	$\theta(X)$			Error	
	NS [8]	HPM[8]	Present work	HPM	Present work
-1.0000	0.5000	0.5000	0.5000	0.0000	0.0000
-0.8000	0.4007	0.4007	0.4007	0.0000	0.0000
-0.6000	0.3011	0.3012	0.3012	0.0001	0.0001
-0.4000	0.2015	0.2016	0.2015	0.0001	0.0000
-0.200	0.1018	0.1019	0.1017	0.0001	0.0001
0.2000	-0.0981	-0.0981	-0.0980	0.0000	0.0000
0.4000	-0.1984	-0.1984	-0.1984	0.0000	0.0000
0.6000	-0.2989	-0.2988	-0.2989	0.0001	0.0000
0.8000	-0.3993	-0.3993	-0.3994	0.0000	0.0001
1.0000	-0.5000	-0.5000	-0.5000	0.0000	0.0000

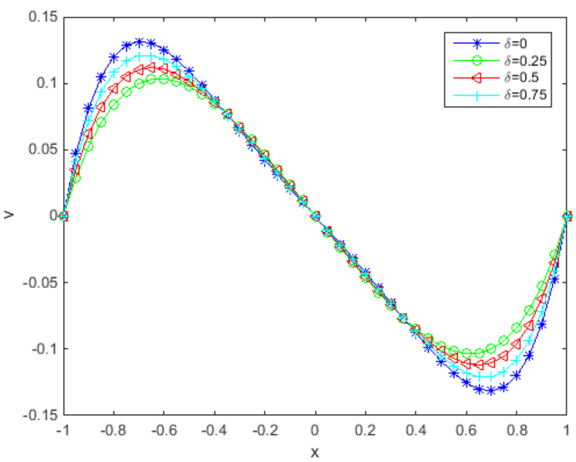

Fig. 2. Effect of non-Newtonian parameter (δ) on velocity profile using Ag when $Ec=\gamma=S=Q=1$ and $\phi=0.01$.

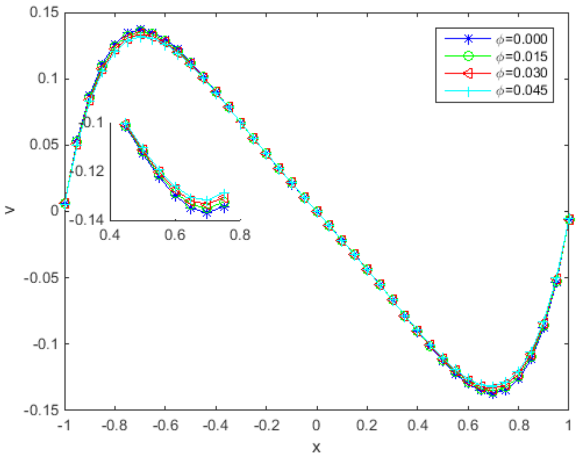

Fig. 3. Effect of Ag nanoparticles concentration (ϕ) on velocity profile when $Ec=S=\gamma=Q=\delta=1$.

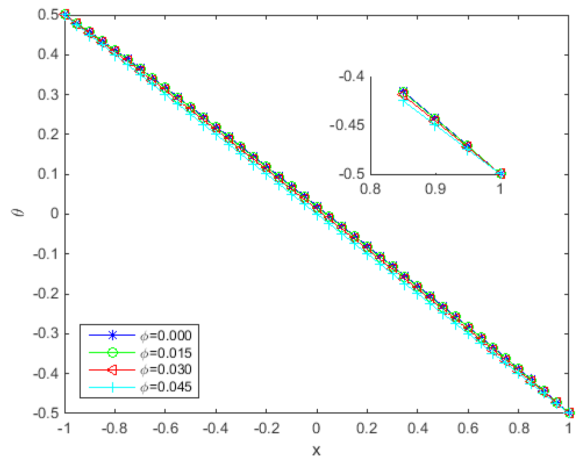

Fig. 4. Effect of Ag nanoparticles concentration (ϕ) on temperature profile when $Ec=S=\gamma=Q=\delta=1$.

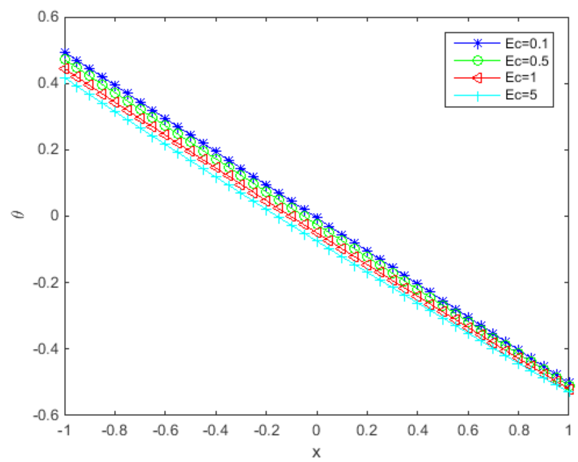

Fig. 5. Effect of Eckert number parameter (Ec.) on temperature profile using Ag when $\delta=Q=\gamma=S=1$ and $\phi=0.01$.

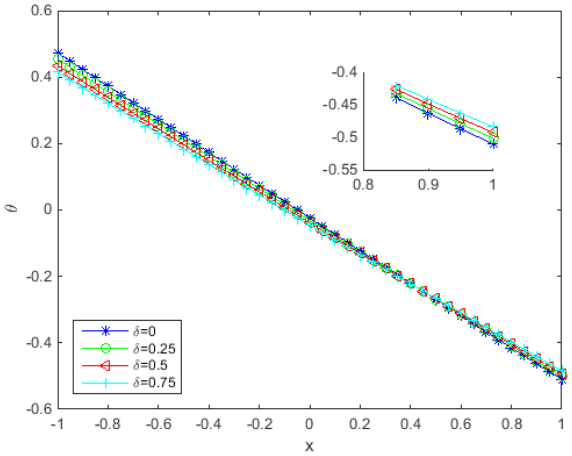

Fig. 6. Effect of non-Newtonian parameter (δ) on temperature profile using Ag when $Ec=\gamma=S=Q=1$ and $\phi=0.01$.

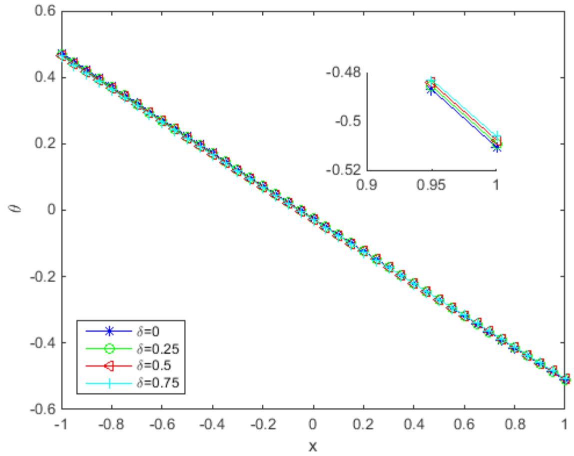

Fig. 9. Effect of non-Newtonian parameter (δ) on temperature profile using Al_2O_3 when $Ec=\gamma=S=Q=1$ and $\phi=0.01$.

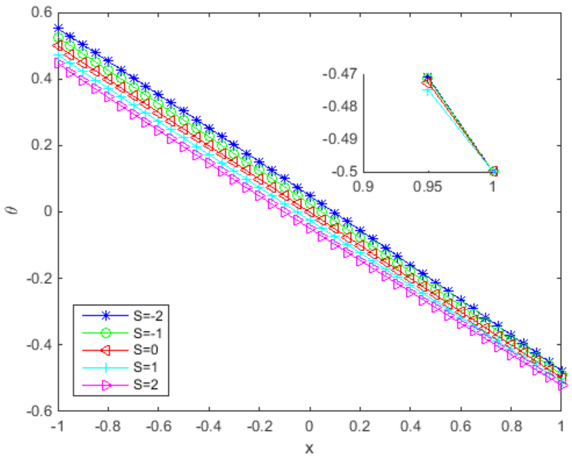

Fig. 7. Effect of Squeezing parameter (S) on temperature profile using Ag when $Ec=\gamma=Q=\delta=1$ and $\phi=0.01$.

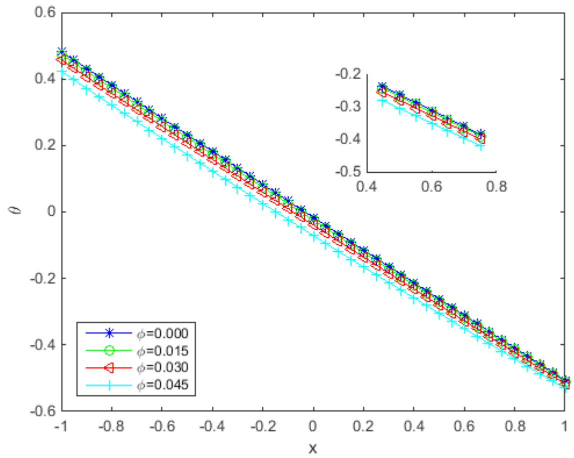

Fig. 10. Effect of Al_2O_3 nanoparticle concentration (ϕ) on temperature profile when $Ec=\gamma=S=Q=\delta=1$.

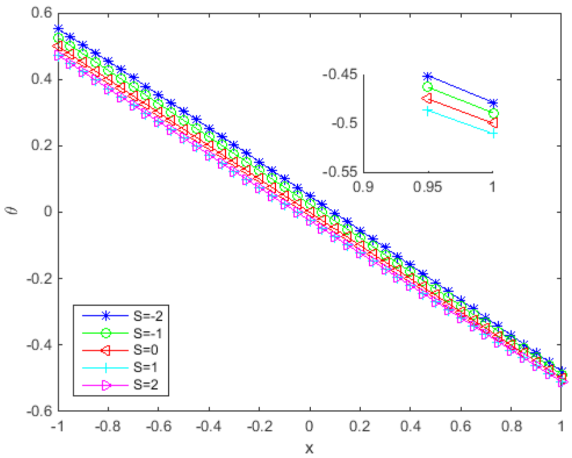

Fig. 8. Effect of Squeezing parameter (S) on temperature profile using Al_2O_3 when $Ec=\gamma=Q=\delta=1$ and $\phi=0.01$.

Nanoparticle concentration effect is presented in the Fig. 3, as observed increasing numeric value of ϕ causes slight increase in velocity distribution which can be physically explained owing to energy improvement exchange rate and heat dissipation. Fig. 4 illustrates nanoparticle concentration (ϕ) effect on temperature distribution, as shown increase in ϕ causes decreasing temperature distribution due to increasing shear at the walls. . Influence of Eckert's number (*Ec.*) on the temperature profile is observed in the Fig. 5, which indicates that temperature decreases slightly at increasing values of Ec. this is caused by increase in fluid thermal energy consequently overall heat transfer. The effect of non-Newtonian parameter (δ) on heat transfer is depicted from Fig. 6, where it is shown that as the fluid becomes more non-Newtonian temperature profile increases slightly. This may be physically explained due to thermal boundary layer thickness decrease.

Fig. 7 illustrates the effect of squeeze on heat transfer rate adopting silver nanoparticles. It is demonstrated from the Fig. 7 that as the plate comes together (S<0) the temperature distribution decreases but as the plates moves apart the temperature distribution further decreases due to varying kinematic viscosity effect on heat exchange. More so utilizing alumina nanoparticle squeezing effect on heat transfer is observed in the Fig. 8 which demonstrates similar decrease in heat transfer rate for plates coming together ($S<0$) and plates moving apart ($S>0$). However the silver nanoparticles shows rapid rate of heat transfer compared with alumina nanoparticle. Utilizing nanoparticles of alumina (Al_2O_3) on the sodium alginate based fluid. It can be demonstrated from the Fig. 9 that as fluid becomes more non-Newtonian with increasing δ, increase in thermal boundary layer is observed. Similarly the effect of alumina nanoparticle concentration (ϕ) on the sodium alginate is demonstrated in Fig. 10 where increasing ϕ leads to decrease in temperature distribution caused by decrease in thermal boundary layer thickness. It is observed from analysis that addition of nanoparticles into base fluid causes slight increase in thermal boundary layer thickness, though effect is insignificant on velocity boundary layer. When nanoparticles are added to base fluid heat transfer increases due to higher conductivity of nanoparticles resulting to decreased temperature distribution. Consequently base fluid with nanoparticles of silver having higher thermal conductivity has lower temperature distribution compared with Alumina with lower thermal conductivity due to increased heat transfer rate; this is illustrated in Table 4.

Table 4. Comparative values of temperature distribution for Prandtl number (*Pr*) using silver and Alumina nanoparticles. When $\delta=Ec=0.5$, $S=1$, $\phi=0.05$.

Pr	Silver(Ag)	Alumina(Al_2O_3)
0.6	0.0067	0.0067
0.8	0.0089	0.0090
1.2	0.0134	0.0135
1.4	0.0156	0.0157
1.6	0.0178	0.0179
1.8	0.0200	0.0202
2.0	0.0202	0.0205

4- Conclusion

In this paper, the squeezing flow on heat transfer of a non-Newtonian sodium alginate (SA) is presented comparing the effect of nanoparticles of silver (Ag) and alumina (Al_2O_3) on heat transfer. The mechanics describing the heat transfer and flow are nonlinear, higher order equations; therefore the ADM has been used to obtain analytical solutions. The approximate analytical solution is used to investigate the behavior of important rheological fluid parameters. Results obtained depicts the silver nanoparticle demonstrate higher heat transfer rates compared to alumina for plates coming together and moving apart. Also the effect of parameters such nanoparticle concentration and non-Newtonian parameter using both silver and alumina were established which proves silver having higher thermal conductivity has lower temperature distribution compared with Alumina due to increased heat transfer rate. The results obtained can be used to advance the study of sodium alginate in processes such as manufacturing and biomedical application.

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Nomenclature

E_c	Eckert number
g	Acceleration due to gravity
G	Material constant
k	Thermal conductivity
k_f	Base fluid thermal conductivity
k_{nf}	Effective thermal conductivity
k_s	Nanoparticle thermal conductivity
Pr	Prandtl number
S	Squeeze parameter
T	Initial temperature
T_m	Mean temperature
v	Velocity in x direction
V	Dimensionless velocity in x direction

Greek symbol

α	Thermal diffusivity
β_3	Activation energy parameter
δ	Dimensionless non-Newtonian parameter
μ_{nf}	Effective dynamic viscosity
θ	Dimensionless temperature
$(\rho C_p)_f$	Base fluid heat capacity
$(\rho C_p)_{nf}$	Effective heat capacity
$(\rho C_p)_s$	Nanoparticle heat capacity
ρ_f	Base fluid density
ρ_{nf}	Effective density
ρ_s	Nanoparticle density
υ	Kinematic viscosity

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