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# The Effect of Concentrated Axial Harmonic Force on the Response of Lateral Vibration for a Jeffcott Rotor

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**ABSTRACT:** Rotors are widely used in industry and studying their vibrations is important. Lateral vibration of the rotors during operation is more important than its other vibration modes such as axial and torsional. The aim of this paper is to determine the effects of loads axially exerted on the assembled disk on a rotor as an introduction to modeling common phenomena such as surge and chock in rotors. Therefore, in this paper, the effect of a concentrated axial force acted on disk on the lateral vibration of a jeffcott rotor is investigated. Also, the effect of unbalance force on vibration behavior of the rotor is studied. The equation of motion was derived from Timoshenko beam model. The set of governing equations for vibration analysis of the rotor consist of four coupled partial differential equations. Since the derived equations are complex and coupled, and they have time-varying coefficients, they are solved by a combination of Galerkin and Newmark methods. Numerical examples are analyzed. The accuracy of derived equations is verified for a simple beam. Results show that the axial load is considerably effective on the amplitude of the lateral vibration of the rotor.

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## 1- Introduction

Rotor Dynamics is a branch of dynamic systems dealing with significant angular momentum. Rotors are widely used in engineering. The rotational motion of rotors, associated with a useful work it is supposed to accomplish, is accompanied by mechanical side effects. Due to several factors, which contribute to the energy transfer, the rotor rotation may be accompanied by various modes of vibrations. All three main modes of rotor vibrations are lateral, torsional and axial modes that may be present during rotor operation. Among these modes, the lateral modes of the rotor are of the greatest concern [1]. The simplest model that can be used to study the flexible behavior of rotors consists of a point mass attached to a massless shaft. It is often referred to as a jeffcott rotor. Rotors of turbines and compressors carry one or more disks. These disks, furthermore, contribute to a lumped mass at stators where they are located. When flexible rotors are investigated, it is a common practice to neglect axial forces and the interaction between these forces and radial vibration. Frequently, axial forces are indeed very small and need to be considered, but for example, in aircraft engines, the thrust is a major axial force in that rotor system. Such axial forces may be constant. But especially in rotating machinery, they may also be harmonic or random.

Nelson et al. [2] studied the vibration analysis of the Timoshenko rotor with an internal damping under an axial load. Edney et al. [3] proposed a dynamic analysis of the tapered Timoshenko rotor. Chen et al. [4] analyzed the exact and direct modeling technique for rotor bearing system subjected to axial load. Choi et al. [5] presented the consistent derivation of a set of governing differential equations describing the vibration in two orthogonal planes and the torsional vibration of a straight rotor with dissimilar lateral principle moments of inertia, subjected to a constant compressive axial load. Huajian gouyang et al. [6] presented a dynamic model for the vibration of a rotating Timoshenko beam subjected to a three-directional load moving in the axial direction. Askarian et al. [7] investigated the effect of various parameters such as axial force, unbalance, and coupling misalignment on the vibration of a rotor. Hosseini et al. [8] analyzed the effect of an axial force and shaft characteristics on the lateral natural frequencies of a flexible rotating shaft. Nawal H.Al et al. [9] derived the equation of motion that governs the transverse vibration of a beam loaded axially and compared the natural frequencies. Motallebi et al. [10] utilized a homotopy analysis based method to consider the vibration of a nonlinear beam under axial force. Torabi et al. [11] presented an analytical solution for the whirling analysis of axial-loaded Timoshenko rotor and corresponding basic function.

In this paper, the dynamics of a rotating beam modeled as a Timoshenko rotor, subjected to concentrated harmonic loads acting on the surface of disk on the rotor and a dynamic model is presented for the vibration of a rotating shaft as a Timoshenko rotor subjected to an axial concentrated load that acts on the surface of disk on the rotor. The equation of motion was derived from Timoshenko beam model, and solved by using Galerkin- Newmark method.

## 2- Governing Equations and Solution Procedure

A simply supported Timoshenko rotor subjected to the axial force acted on disks is shown in Fig.1.

As shown in Fig. 2¸ the concentrated load can introduce the bending moment, below:

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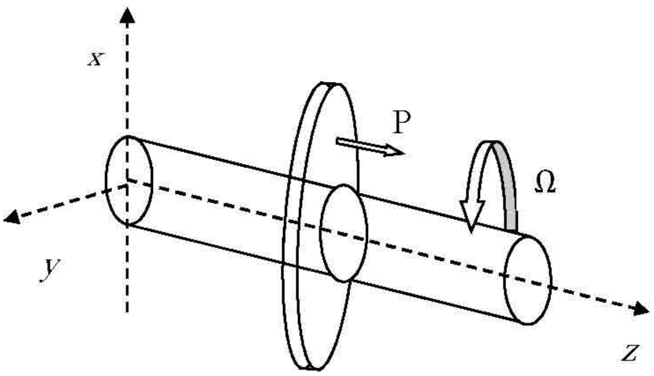


**Fig. 1. Rotating Timoshenko shaft with an axial loaded disk**

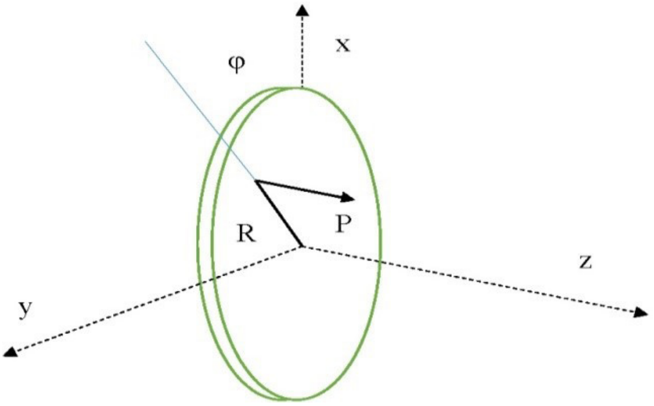


**Fig. 2. Concentrated load position on the disk**

$$\begin{aligned} m_x &= PR\cos\varphi\,\delta(z - z_0) \\ m_y &= PR\sin\varphi\,\delta(z - z_0) \end{aligned} \tag{1}$$

By using the equilibrium of forces and momentums, the set of equations of motion can be extracted as follows:

$$\begin{aligned}
&F_x + \frac{\partial F_x}{\partial z}dz - F_x + \left(P + \frac{\partial P}{\partial z}dz\right)\left(\frac{\partial u_x}{\partial z} + \frac{\partial^2 u_x}{\partial z^2}dz\right) \\
&-P\frac{\partial u_x}{\partial z} + f_x(x,t) = \left[\rho A + M_0\delta(z - z_0)\right]dz\,\frac{\partial^2 u_x}{\partial t^2} \\
&F_y + \frac{\partial F_y}{\partial z}dz - F_y + \left(P + \frac{\partial P}{\partial z}dz\right)\left(\frac{\partial u_y}{\partial z} + \frac{\partial^2 u_y}{\partial z^2}dz\right) \\
&-P\frac{\partial u_y}{\partial z} + f_y(x,t) = \left[\rho A + M_0\delta(z - z_0)\right]dz\,\frac{\partial^2 u_y}{\partial t^2} \\
&M_x + \frac{\partial M_x}{\partial z}dz - M_x - \left(F_y + \frac{\partial F_y}{\partial z}dz\right)dz + m_x = \\
&\left[\rho I_x + I_x\delta(z - z_0)\right]\frac{\partial^2 \varphi_x}{\partial t^2}dz \\
&+\left[\rho I_p + I_p\delta(z - z_0)\right]\Omega dz\,\frac{\partial \varphi_y}{\partial t} \\
&M_y + \frac{\partial M_y}{\partial z}dz - M_y - \left(F_x + \frac{\partial F_x}{\partial z}dz\right)dz + m_y = \\
&\left[\rho I_y + I_y\delta(z - z_0)\right]\frac{\partial^2 \varphi_y}{\partial t^2}dz \\
&+\left[\rho I_p + I_p\delta(z - z_0)\right]\Omega dz\,\frac{\partial \varphi_x}{\partial t}
\end{aligned} \tag{2}$$

Where $u_x$ , $u_y$ , $\varphi_x$ and $\varphi_y$ are components of displacement and rotation in $x$ and $y$ directions, respectively; $\rho$ is mass density. In addition A, $I_x$ , $I_y$ and $I_p$ are cross- sectional area, moment of inertia about the x and y axes and polar moment of inertia, respectively; $f_x$ and $f_y$ and $P$ are forces per unit length in $x$ and $y$ directions and axial force.

According to Timoshenko beam theory, components of bending moment ($M$) and shear force ($F$) in $x$ and $y$ directions are presented as follows [12]:

$$\begin{aligned}
F_x &= kGA\left(\frac{\partial u_x}{\partial z} - \varphi_y\right) \\
F_y &= kGA\left(\frac{\partial u_y}{\partial z} - \varphi_x\right) \\
M_x &= EI_x\frac{\partial \varphi_x}{\partial z} \\
M_y &= EI_y\frac{\partial \varphi_y}{\partial z}
\end{aligned} \tag{3}$$

Were $E$ and $G$ are the modulus of elasticity and shear modulus, respectively; $k$ is the shear correction factor depending on the shape of the section and Poisson's ratio of material [13]. By neglecting the term of $(dz)^2$ and using the following relation for a circular section

$$I_p = 2I_x = 2I_y = 2I \tag{4}$$

Eq.(2) can be written as:

$$\begin{aligned}
&kAG\left(\frac{\partial^2 u_x}{\partial z^2} - \frac{\partial \varphi_y}{\partial z}\right) + \frac{\partial}{\partial z}\left(P\frac{\partial u_x}{\partial z}\right) + f_x(x,t) = \\
&\left[\rho A + M_0\delta(z - z_0)\right]\frac{\partial^2 u_x}{\partial t^2} \\
&kAG\left(\frac{\partial^2 u_y}{\partial z^2} - \frac{\partial \varphi_x}{\partial z}\right) + \frac{\partial}{\partial z}\left(P\frac{\partial u_y}{\partial z}\right) + f_y(x,t) = \\
&\left[\rho A + M_0\delta(z - z_0)\right]\frac{\partial^2 u_y}{\partial t^2} \\
&EI\frac{\partial^2 \varphi_x}{\partial Z^2} - kGA\left(\frac{\partial u_y}{\partial Z} + \varphi_x\right) + m_x = \\
&\left[\rho I + I\delta(z - z_0)\right]\frac{\partial^2 \varphi_x}{\partial t^2} + 2\left[\rho I + I_0\delta(z - z_0)\right]\Omega\frac{\partial \varphi_y}{\partial t} \\
&EI\frac{\partial^2 \varphi_y}{\partial z^2} - kGA\left(\frac{\partial u_x}{\partial z} + \varphi_y\right) + m_y = \\
&\left[\rho I + I_0\delta(z - z_0)\right]\frac{\partial^2 \varphi_y}{\partial t^2} + 2\left[\rho I + I_0\delta(z - z_0)\right]\Omega\frac{\partial \varphi_x}{\partial t}
\end{aligned} \tag{5}$$

The force at the cross-section of the rotor is written as:

$$P(z,t) = -R_0 - P_0(t)H(z - z_0) \tag{6}$$

By re-arranging, the equations of motion can be written:

$$kGA\left(\frac{\partial^2 u_x}{\partial z^2}+\frac{\partial \varphi_x}{\partial z}\right)-\left[R_0+P_0(t)H(z-z_0)\right]\frac{\partial^2 u_x}{\partial z^2}$$
$$-P_0(t)H(z-z_0)\frac{\partial u_x}{\partial z}+f_x(x,t)=$$
$$\left[\rho A+M_0\delta(z-z_0)\right]\frac{\partial^2 u_x}{\partial t^2}$$
$$kGA\left(\frac{\partial^2 u_y}{\partial z^2}+\frac{\partial \varphi_y}{\partial z}\right)-\left[R_0+P_0(t)H(z-z_0)\right]\frac{\partial^2 u_y}{\partial z^2}$$
$$-P_0(t)\delta(z-z_0)\frac{\partial u_y}{\partial z}+f_y(x,t)=$$
$$\left[\rho A+M_0\delta(\mathrm{z}-\mathrm{z}_0)\right]\frac{\partial^2 u_y}{\partial t^2} \tag{7}$$
$$EI\frac{\partial^2 \varphi_x}{\partial z^2}-kGA\left(\frac{\partial u_y}{\partial z}+\varphi_x\right)$$
$$+PRcos\varphi\delta(z-z_0)=\left[\rho I+I_0\delta(\mathrm{z}-\mathrm{z}_0)\right]\frac{\partial^2 \varphi_x}{\partial t^2}$$
$$+2\left[\rho I+I_0\delta(\mathrm{z}-\mathrm{z}_0)\right]\Omega\frac{\partial \varphi_y}{\partial t}$$
$$EI\frac{\partial^2 \varphi_y}{\partial z^2}+kGA\left(\frac{\partial u_x}{\partial z}-\varphi_y\right)+PRsin\varphi\delta(z-z_0)=$$
$$\left[\rho I+I_0\delta(\mathrm{z}-\mathrm{z}_0)\right]\frac{\partial^2 \varphi_y}{\partial t^2}+2\left[\rho I+I_0\delta(\mathrm{z}-\mathrm{z}_0)\right]\Omega\frac{\partial \varphi_x}{\partial t}$$

By introducing the following complex variables ($J^2 = -1$)

$$u=u_x+ju_y$$
$$\varphi=\varphi_x+j\varphi_y \tag{8}$$
$$f=f_x+jf_y$$

The external forces that include weight and unbalanced force can be written as:

$$f(x,t)=f_1(x,t)+f_2(x,t)=\rho Ag-$$
$$[M_0 g\delta(z-z_0)+\Omega^2 m_0^p\left[\cos(\Omega t+\theta_0)\right]\delta(z-z_0)] \tag{9}$$

Thus, the set of equations of motion can be represented in the following forms:

$$-\left[\rho A+M_0\delta(z-z_0)\right]\frac{\partial^2 u}{\partial t^2}+kGA\left(\frac{\partial^2 u}{\partial z^2}+j\frac{\partial \varphi}{\partial z}\right)$$
$$-\left[R_0+P_0(t)H(z-z_0)\right]\frac{\partial^2 u}{\partial z^2}-P_0(t)\delta(z-z_0)\frac{\partial u}{\partial z}$$
$$=\rho Ag+\left[P_0 M_0 g-m_0^p\Omega cos(\Omega t+\theta_0)\right]\delta(z-z_0)$$
$$-j\Omega m_0^p sin(\Omega t+\theta_0)\delta(z-z_0) \tag{10}$$
$$-\left[\rho I+I_0\delta(z-z_0)\right]\frac{\partial^2 \varphi}{\partial t^2}$$
$$+2j\left[\rho I+I_0\delta(z-z_0)\right]\Omega\frac{\partial \varphi}{\partial t}+EI\frac{\partial^2 \varphi}{\partial z^2}$$
$$-kGA\left(\varphi-j\frac{\partial u}{\partial z}\right)=PRe^{-J\varphi}\delta(Z-Z_0)$$

For simply supported rotor, it can be written:

$$z=0:u=0,\frac{\partial \varphi}{\partial z}=0$$
$$z=L:u=0,\frac{\partial \varphi}{\partial z}=0 \tag{11}$$

Thus, the solution can be assumed:

$$u(z,t)=\sum_{n=1}^{\infty}a_n(t)\sin(\frac{n\pi z}{L})$$
$$\varphi(z,t)=\sum_{n=1}^{\infty}b_n(t)\cos\left[\frac{(n-1)\pi z}{L}\right] \tag{12}$$

Inserting Eq.(12) into Eq.(10) and then doing the required simplification, the below equations are obtained:

$$\sum_{n=1}^{\infty}\left\{\begin{array}{c}\left[\rho A+M_0\delta(z-z_0)\right]sin\left(\frac{n\pi z}{L}\right)\ddot{a}_n(t)\\ +\left\{\begin{array}{c}\left(\frac{n\pi}{L}\right)^2\left[kGA-R_0-P_0(t)H(\mathrm{z}-\mathrm{z}_0)\right]sin\left(\frac{n\pi z}{L}\right)\\ +\left[P_0(t)\delta(\mathrm{z}-\mathrm{z}_0)\right]\frac{n\pi}{L}cos\left(\frac{n\pi z}{L}\right)\end{array}\right\}\\ a_n(t)\\ +jkGA\left[\frac{(n-1)\pi z}{L}\right]sin\left[\frac{(n-1)\pi z}{L}\right]b_n(t)\end{array}\right\}$$
$$=-\rho Ag+\left\{\begin{array}{c}m_0^p e_0 Ù^2\left[\cos(Ùt+è_0)+jsin(Ùt+è_0)\right]\\ -M_0 g\end{array}\right\}$$
$$\delta(z-z_0) \tag{13}$$
$$\sum_{n=1}^{\infty}\left\{\begin{array}{c}\left[\rho I+I_0\delta(z-z_0)\right]\ddot{b}(t)-\\ 2j\Omega\left[\rho I+I_0\delta(\mathrm{z}-\mathrm{z}_0)\right]\dot{b}_n(t)\\ +EI\left[\frac{(n-1)\pi z}{L}\right]^2 b_n(t)+\\ kGAb_n(t)-\\ j\frac{n\pi}{L}kGAa_n(t)cos\left(\frac{n\pi z}{L}\right)\end{array}\right\}\cos\left[\frac{(n-1)\pi z}{L}\right]$$
$$=PRe^{-J\varphi}\delta(Z-Z_0)$$

By applied Galerkin method can be written:

$$\sum_{n=1}^{\infty}\int_0^L \left\{ \begin{array}{c} \left[ \begin{array}{c} \rho A+\sum_{i=1}^{2} M_0\delta(z-z_0) \\ sin\left(\frac{n\pi z}{L}\right) sin\left(\frac{m\pi z}{L}\right)\ddot{a}_n(t) \end{array} \right] + \\ \left[ \begin{array}{c} \left(\frac{n\pi}{L}\right)^2 \left[kGA-R_0-P_0H(z-z_0)\right] \\ sin\left(\frac{n\pi z}{L}\right) sin\left(\frac{m\pi z}{L}\right) \\ +P_0\delta(z-z_0)\frac{n\pi}{L} \\ cos\left(\frac{n\pi z}{L}\right) sin\left(\frac{m\pi z}{L}\right) \end{array} \right] \\ a_n(t) \\ +jkGA\frac{(n-1)\pi}{L} sin\left(\frac{(n-1)\pi z}{L}\right) \\ sin\left(\frac{m\pi z}{L}\right) b_n(t) \end{array} \right\} dz$$

$$=-\rho A g\int_0^L sin\left(\frac{m\pi z}{L}\right)dz \tag{14}$$

$$+\left\{m_0^p e_0\Omega^2\left[\cos(\Omega t+\theta_0)+jsin(\Omega t+\theta_0)\right]-M_0 g\right\}$$

$$\int_0^L \delta(z-z_0)\sin\left(\frac{m\pi z}{L}\right)dz$$

$$\sum_{n=1}^{\infty}\int_0^L \left\{ \begin{array}{c} \left[\rho I+I_0\delta(z-z_0)\right]\ddot{b}(t) \\ -2j\Omega\left[\rho I+I_0\delta(z-z_0)\right]\dot{b}_n(t) \\ +EI\left(\frac{(n-1)\pi}{L}\right)^2 b_n(t) \\ +kGA\, b_n(t)-jkGA\frac{n\pi}{L}a_n(t)cos\left(\frac{n\pi z}{L}\right) \end{array} \right\}$$

$$\cos\left[\frac{(m-1)\pi z}{L}\right]dz=PRe^{-J\varphi}\int_0^L cos\left(\frac{(m-1)\pi z}{L}\right)$$

$$\delta(Z-Z_0)dz$$

$$[M]\{\ddot{X}(t)\}+[C]\{\dot{X}(t)\}+[K(t)]\{X(t)\}=[F(t)]$$

The simplified form can be summarized as:

$$[M]\{\ddot{X}(t)\}+[C]\{\dot{X}(t)\}+[K(t)]\{X(t)\}=[F(t)] \tag{15}$$

## 3- Numerical Results and Discussion

In order to analyze the results, having the specifications shown in Table 1¸ a rotor was chosen. In order to examine the accuracy of the drawn up codes, the rotor was first modeled as a simple beam without rotation, while the results were compared with the accurate solution to the problem. The results were then obtained through assuming $\alpha$=1/2, $\beta$=1/4 and $\Delta t$=0.01 in Newmark-beta solution [14] and by taking into accounts the first five sentences of Galerkin solution ($N_i$=5) by coding in the MATLAB software.

**Table 1. Considered rotor characteristics**

| Characteristics | Value | Symbols & dimensions |
|---|---|---|
| Rotor diameter | 50 | $d$ (mm) |
| Rotor length | 4 | $L$ (m) |
| Modulus of elasticity | 200 | $E$ (GPa) |
| Poisson's ratio | 0.3 | $\upsilon$ |
| Density | 7860 | $\rho$ (Kg/m$^3$) |
| Location of disk | $L$/2 | $z_0$ |
| Mass of disk | 5 | $M_0$ (Kg) |
| Moment of inertia | 2 | $I_0$ (Kg.m$^2$) |
| Unbalance mass | 10 | $m_0^p$ (gr) |
| Unbalance radios | 50 | $e_0$ (mm) |
| Unbalance angle with x direction | 45 | $\theta_0$ (degree) |
| Rotational speed | 50 | $\Omega$ (rpm) |

Given that there is no reference to verify the results, modeling method and the drawn up codes, the axial force, and the rotor's rotation were removed; and merely a model of the beam with a simple support was taken into account. Therefore, the extent of motion at the left support and the shear force along x axis in the left support of the rotor were calculated and shown in Figs.3 and 4.

The results revealed that a constant force of approximately 284 Newton is exerted on the left support. The extent of motion at the rotor's left support is also zero. Table 2 compares and shows the error percentage among values resulting from the present study and their analytical solutions. The results verify the method and the drawn up codes.

**Table 2. Comparison between the present method and the analytical results**

| Parameter | Analytical solution | Present solution | Error percentage |
|---|---|---|---|
| $F_x$, N | 301 | 284 | 5.7 |

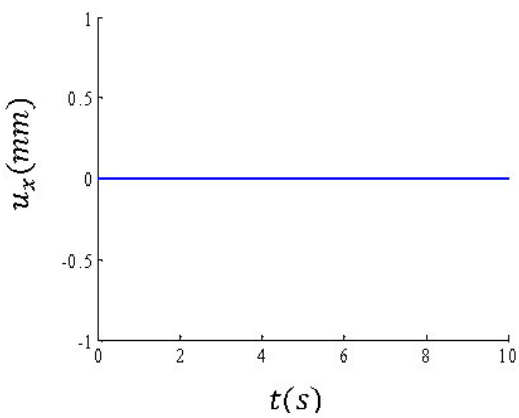


Fig. 3. Displacement (x direction)

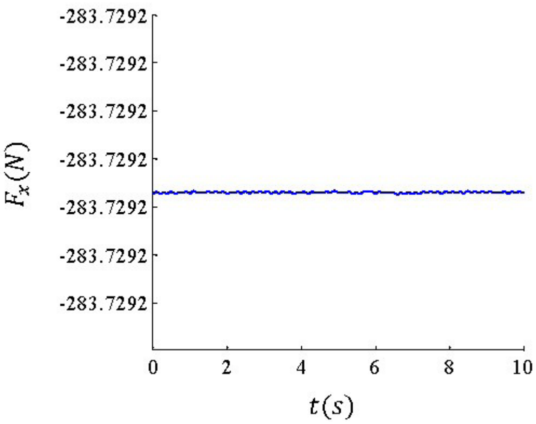


**Fig. 4. Shear force (x direction)**

In order to study the effects of the axial load on the responses, a harmonic axial load on the disk is considered whose amplitude and frequency are 20 kN and 5 rad/s.

Fig.5 shows the results at the left support for the unbalanced rotor with an axial harmonic force ($p$=20sin(5$t$)).

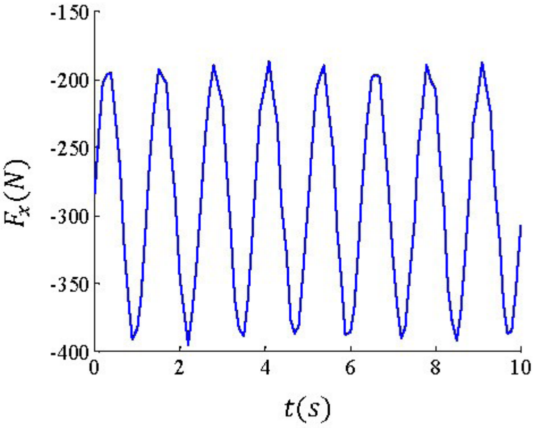


**Fig. 5. Shear force (x direction)**

As shown in these figure¸ the value of the $x$ direction shear force increases.

Also, for the unbalanced rotor with an axial harmonic force ($p$=100sin(5$t$)) results at the left support are depicted in Fig.6. The amplitude increasing $x$ direction shear force can be easily seen in this figure.

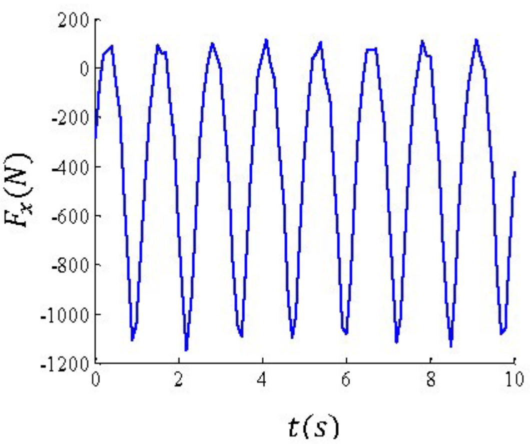


**Fig. 6. Shear force (x direction)**

## 4- Conclusions

The effect of axial concentrated forces acted on the disk on the lateral vibration of a modified jeffcott rotor was analyzed. The rotor was assumed to be uniform and Timoshenko theory was used. Partial differential equations of motion were derived by considering equilibrium equations for an element of the rotor. The rotor was considered simply supported and Galerkin-Newmark method was applied directly to the partial differential equations of motion. Responses of the lateral vibration with and without axial force were analyzed. From the results, it is observed that by increasing the amplitude of the force the response increases. The results could be used for the analysis of the phenomena , including surge and stall in the gas turbines in order to predict them. Therefore, if the forces at supports are measured and the changes are taken into account, these phenomena could be identified.

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